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Representations of the ecological niche

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Abstract. A formal theory of the ecological niche is indispensable not only for semantic precision in philosophy to understand and compare it with other meanings of niche, but also when computer scientists and ecologists desire to create interoperable software where one can retrieve the niche of a species and compare their parameters. The proposed model is a more fine-grained description of the ecological niche, including the distinction between its complex concept, the abstract niche ('fundamental niche') with its hypervolume in multidimensional space, and its realisations ('realised niches'). The presented ecological niche may initiate new avenues for research in ecology, particularly concerning the conditions/categories of a hypervolume, as well as further philosophical inquiry and comparison with other niches.

1 Introduction

The ecological niche is a fascinating complex concept and receives attention from people with rather diverse backgrounds: philosophers, bioinformaticians and computer scientist, and, of course, ecologists. Each of them may emphasise different aspects of the ecological niche, use different established vocabularies, and has their preferred representation. In this appendix we show the main components in Object-Role Modeling (ORM) representation and its verbalization in section 2 and the First Order Logic representation in section 3.

2 ORM representation of the ecological niche

To meet differences in preferences of representing knowledge, we have modelled the main components of the ecological niche in the Object-Role Modelling (ORM) conceptual modelling method. Two important advantages are that, first, the model is a *conceptual model* and its modeling tools, like VisioModeler 3.1 and Infagon, have software supported design and implementation transformations of ORM conceptual models into databases, among others; refer to [1] for a comprehensive explanation of ORM and mappings to UML and ER. Second, the tools have a nice feature called *verbalization*, which renders the diagrammatic representation into fixed-syntax pseudo-natural language sentences. Although the automatically generated sentences are not always grammatically perfect, they are understandable for domain experts who are unfamiliar with the diagrammatic representation and axiomatizations. DogmaModeler, an ontology engineering tool based on ORM, supports verbalization not only in English, but also in nine other languages, including Italian, Spanish, German, Russian, Arabic, and Dutch [2]. The verbalization included here is in English only.

The ORM model is depicted in Fig.1, with corresponding verbalization of the facts afterwards.

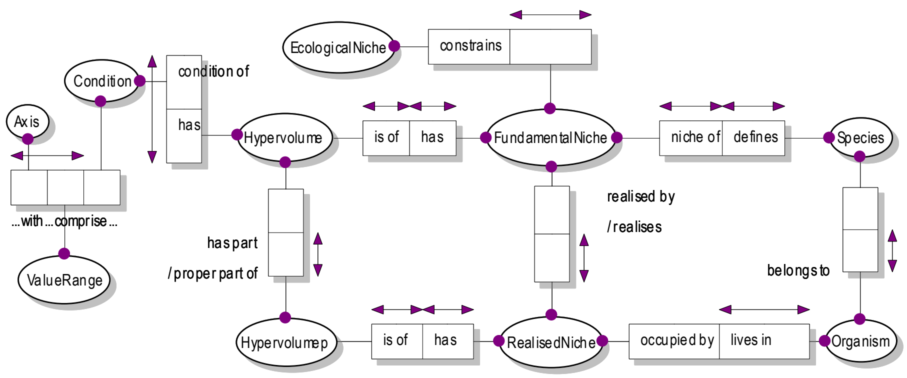

Fig. 1. The ORM representation of the ecological niche shows graphically the main components of the ecological niche, where ovals denote entities, rectangles relations, arrows uniqueness constraints, and blobs mandatory constrains (included in the formalisation).

2.1 Verbalization of the ORM model of the ecological niche

EcologicalNiche constrains FundamentalNiche

Each EcologicalNiche constrains **some** FundamentalNiche.

For each FundamentalNiche **f, some** EcologicalNiche constrains FundamentalNiche **f**.

For each FundamentalNiche **f, at most one** EcologicalNiche constrains FundamentalNiche **f**.

Species defines FundamentalNiche **/** FundamentalNiche niche of Species.

Each FundamentalNiche niche of **some** Species.

Each FundamentalNiche niche of **at most one** Species.

Each Species defines **some** FundamentalNiche.

Each Species defines **at most one** FundamentalNiche.

FundamentalNiche has Hypervolume **/** Hypervolume is of FundamentalNiche.

Each FundamentalNiche has **some** Hypervolume.

Each FundamentalNiche has **at most one** Hypervolume.

Each Hypervolume is of **at most one** FundamentalNiche.

Condition condition of Hypervolume **/** Hypervolume has Condition.

Each Condition condition of **some** Hypervolume.

Each Hypervolume has **some** Condition.

It is possible that some Condition condition of **more than one** Hypervolume
and that some Hypervolume has **more than one** Condition.

Axis with ValueRange comprise Condition **/ each** Axis with **some** ValueRange comprise **some** Condition.

For each ValueRange **v, some** Axis with ValueRange **v** comprise **some** Condition.

For rach Condition **c, some** Axis with **some** ValueRange comprise Condition **c**.

Given any Axis **and** ValueRange **, that** Axis with **that** ValueRange comprise **at most one** Condition.

FundamentalNiche realised by RealisedNiche **/** RealisedNiche realises FundamentalNiche.

Each FundamentalNiche realised by **some** RealisedNiche.

Each RealisedNiche realises **some** FundamentalNiche.

Each RealisedNiche realises **at most one** FundamentalNiche.

Hypervolumep proper part of Hypervolume **/** Hypervolume has part Hypervolumep.

Each Hypervolumep proper part of **some** Hypervolume.

Each Hypervolume has part **some** Hypervolumep.

Each Hypervolumep proper part of **at most one** Hypervolume.

Organism belongs to Species.

Each Organism belongs to **some** Species.

For each Species **s, some** Organism belongs to Species **s**.

Each Organism belongs to **at most one** Species.

Organism lives in RealisedNiche **/** RealisedNiche occupied by Organism.

Each Organism lives in **some** RealisedNiche.

Each RealisedNiche occupied by **some** Organism.

Each Organism lives in **at most one** RealisedNiche.

RealisedNiche has Hypervolumep **/** Hypervolumep is of RealisedNiche.

Each RealisedNiche has **some** Hypervolumep.

Each RealisedNiche has **at most one** Hypervolumep.

Each Hypervolumep is of **at most one** RealisedNiche.

3 Formalisation

The abbreviations used here, including the ones that adhere to the DOLCE foundational ontology [4] [5], are provided in §3.1. Here, I do not concern about the entity vs. concept issue: both DOLCE and ecologists use the concept-standpoint, even though the intention is to represent the intrinsic nature and not the 'lens', and especially ecologists (ought to) have a revisionary attitude where linguistic expressions can be changed/reinterpreted if it is not defensible on scientific grounds. That DOLCE is intended for descriptive ontology does not have to prevent one to use it for representing scientific knowledge. This research and formalisation is of an applied nature and not intended to create another top-level ontology but to exploit an existing one that is well documented, formalised, and relatively understandable. In addition, it already has been applied successfully to other aspects in ontology development for ecology [3].

3.1 Conventions and abbreviations

The formalisation of the ecological niche takes advantage of the formalised DOLCE foundational model. Several notions are directly used in the axioms and therefore briefly listed here; more detail with its machinery can be found in [4] [5]:

- $CN(x)$ stands for that x is a (social) concept, where $CN(x) \rightarrow NASO(x)$ in DOLCE and $NASO$ is the abbreviation for 'non-agentive social object', thus CN is an endurant. Further, a CN "(i) is not directly located in space and, in general, has no direct spatial qualities (ii) has no intentionality; (iii) depends on a community of intentional agents" [5].
- $DF(x, y)$ is short-hand for $definedBy(x, y)$, and $DF(x, y,) \rightarrow (CN(x) \wedge DS(y))$.
- $Q(x)$ quality. Quality of type ϕ (Dd29 in [4]): $qt(\phi, x, y) \triangleq qt(x, y) \wedge \phi(x) \wedge SBL_X(Q, \phi)$. See Dd8 and Dd11 in [4] for the leaf-subsumption relation SBL, and ϕ is a variable ranging on universals. For direct quality (Dd28 in [4]): $dqt(x, y) \triangleq qt(x, y) \wedge \neg\exists z(qt(x, z) \wedge qt(z, y))$
- $R(x)$ stand for x is a region, and $Q(x)$ has a quale in the region $R(x)$ $ql(x, y) \rightarrow (\alpha R(x) \wedge \alpha Q(y))$, where ql is the quale, R the region and Q the quality and the modifier α that it is not just any type of region with any type of quality, but temporal quality TQ with temporal region TQ, PQ and PR for physical, and so forth; more specific axioms related to qualities, quales and regions are in [4].
- $Set(x)$ for the DOLCE set, which is subsumed by AB (abstract).
- $PRE(x, t)$ for being present at time t (Dd40 in [4]): $PRE(x, t) \triangleq \exists t' ql_T(t', x) \wedge P(t, t')$
- $T(t)$ is a time interval.
- $APO(x)$ says that x is an Agentive Physical Object.

In addition to the DOLCE categories, the entities listed in *Table 1* are used for the formalisation of the ecological niche. Other conventions adopted:

- Constants denoting universals: FuN, CN, ...
- Variables ranging over particulars: u, ..., x, y, z.
- $U(x)$ for a universal (concept/entity). The finite set of universals taken from DOLCE and introduced universals for the ecological niche, $\prod_{EcoNiche} = \{EN, FuN, ReN, Sp, C, DF, CN, PT, DS, Q, R, PC, PRE\}$
- The quantifier $\forall$ is omitted for variables where applicable, unless a numerical restriction is required.

Abbreviation	***Comments***
EN	Ecological Niche
FuN	Fundamental Niche
ReN	Realised Niche
Sp	Species
O	Organism
HV	HyperVolume
C	Condition of a HV
Ax	Axis of a C
$VaRa$	The applicable value range of an Ax

Table 1. Abbreviations and entities introduced for the ecological niche.

- The unique existential quantifier has the usual "!" after the $\exists$, which is shorthand for: $\exists!x\phi \leftrightarrow \exists y \forall x(\phi \leftrightarrow x = y)$,
- Numerical restrictions on quantifiers ranging over variable, such as $\forall^{\geq 2}x$ read as "for all x where there are at least two particulars". Such particulars are subsequently indicated with subscripts $_i$, $_j$, $_1$, $_2$ etc. The 'long' version, indicating a 1:n relationship between two different entities A and B , i.e. for $\exists^{\geq 1}y$, is:
  $A(y) \rightarrow \exists x_1, , x_n(B(x_1) \wedge ... \wedge B(x_n)) \wedge (\neg(x_1 = x_2) \wedge ... \wedge \neg(x_1 = x_n) \wedge ... \wedge \neg(x_{n-1} = x_n)) \wedge n \geq 1 \wedge (relation(x_1, y) \wedge ... \wedge (relation(x_n, y)))$
  This is abbreviated as: $A(y) \rightarrow \exists^{\geq n}x(B(x) \wedge relation(x, y) \wedge n \geq 1)$
  The $\forall$ quantifier has an analogous specification.
- An *exclusive-or* is denoted with an underlined $\vee$, as in $\underline{\vee}$.

3.2 Definitions

$$EN(x) \triangleq \exists y, z \exists^{\geq 1} w(nicheOf(y, z) \wedge realises(w, y) \wedge Living(z) \wedge FuN(y) \wedge S(z) \wedge ReN(w)) \tag{1}$$

$$FuN(x) \triangleq \exists y, z(HV(y) \wedge Living(z) \wedge Sp(z)) \tag{2}$$

$$ReN(x) \triangleq \exists y, z \exists^{\geq 1} w(HV(y) \wedge Living(z) \wedge HV_p(w) \wedge properPartOf(w, y)) \tag{3}$$

$$HV(x) \triangleq \exists^{\geq 1} y(C(y) \wedge \neg(y_i = y_j) \wedge conditionOf(y, x)) \wedge AR(x) \tag{4}$$

$$HV_p(x) \triangleq \exists^{\geq 1} y(C(y) \wedge \neg(y_i = y_j) \wedge conditionOf(y, x)) \wedge AR(x) \wedge properPartOf(x, z) \wedge HV(z) \tag{5}$$

$$C(x) \triangleq (Ax(x) \wedge VaRa(y) \wedge hasValues(x, y)) \tag{6}$$

$$Ax(x) \triangleq (Property(x) \underline{\vee} CN(x)) \tag{7}$$

$$Living(y) \triangleq (instance(x, O, t_{now}) \wedge belongsTo(x, y) \wedge O(x) \wedge Sp(y)) \tag{8}$$

$$Extinct(y) \triangleq (instance(x, O, t') \wedge \neg(instance(x, O, t_{now}) \wedge (t' < t_{now}) \wedge belongsTo(x, y) \wedge O(x) \wedge Sp(y)) \tag{9}$$

3.3 Relations

$$constrains(x,y) \rightarrow ((EN(x) \wedge FuN(y)) \quad (10)$$

$$constrains(x,y) \leftrightarrow constrainedBy(y,x) \quad (11)$$

$$\forall x \exists y(belongsTo(y,x) \rightarrow (Sp(x) \wedge O(y))) \quad (12)$$

$$properPartOf(x,y) \rightarrow (HV_p(x) \wedge HV(y)) \quad (13)$$

$$conditionOf(y,x) \rightarrow (HV(x) \wedge C(y)) \quad (14)$$

$$realises(x,y) \rightarrow (ReN(x) \wedge FuN(y)) \quad (15)$$

$$realisedBy(y,x) \rightarrow (ReN(x) \wedge FuN(y)) \quad (16)$$

$$constrains(y,x) \rightarrow (ReN(x) \wedge FuN(y)) \quad (17)$$

$$constrainedBy(x,y) \rightarrow (ReN(x) \wedge FuN(y)) \quad (18)$$

$$constrainedBy(x,y) \rightarrow (ReN(x) \wedge EN(y)) \quad (19)$$

$$\forall x \exists^{\geq 1} y(elementOf(y,x) \rightarrow (U(y) \wedge Set(x))) \quad (20)$$

$$partOf(x,y) \rightarrow (HV(y) \wedge HV_s(x)) \quad (21)$$

$$has(x,y) \rightarrow (FuN(x) \wedge HV(y)) \quad (22)$$

$$has(x,y) \rightarrow (ReN(x) \wedge HV_p(y)) \quad (23)$$

3.4 Constraints, characteristics, and related issues

General, w.r.t. *EN* and *FuN*

$$EN(x) \rightarrow (CN(x) \wedge NASO(x)) \quad (24)$$

$$FuN(x) \rightarrow (CN(x) \wedge NASO(x)) \quad (25)$$

$$O(x) \rightarrow APO(x) \quad (26)$$

$$Sp(x) \rightarrow NASO(x) \quad (27)$$

$$\forall x \exists^{\geq 1} y(HV(x) \wedge HV_p(y)) \quad (28)$$

$$\exists^{\geq 1} x \exists^{\geq 1} y(HV_p(x) \wedge C(y) \wedge conditionOf(y,x) \wedge conditionOf(y_i,x_i) \wedge conditionOf(y_j,x_j) \wedge (y_i \subset y_j \vee y_j \subset y_i \vee \neg(y_i = y_j)) \quad (29)$$

$$\forall x \forall y \exists^{=m} z \exists^{=n} w((HV(x) \wedge HV_p(y) \wedge C(z) \wedge C(w) conditionOf(z,x) \wedge conditionOf(w,y) \wedge (m \geq n) \wedge (n > 3)) \rightarrow ((w \subset z) \vee \forall u,v(VaRa(u) \wedge VaRa(v) \wedge hasValues(z,u) \wedge hasValues(w,v) \rightarrow (z > u)))) \quad (30)$$

$$\forall x \exists^{\geq 1} y(FuN(y) \wedge ReN(x)) \quad (31)$$

$$EN(x) \rightarrow \exists y,z(FuN(y) \wedge Sp(z) \wedge nicheOf(y,z)) \quad (32)$$

$$(FuN(x) \wedge Sp(y) \wedge nicheOf(x,y_i) \wedge nicheOf(x,y_j)) \rightarrow (y_i = y_j) \quad (33)$$

$$(FuN(x) \wedge Sp(y) \wedge nicheOf(x_i,y) \wedge nicheOf(x_j,y)) \rightarrow (x_i = x_j) \quad (34)$$

$$\neg \exists x(Sp(x) \rightarrow \neg \exists y FuN(y) \quad (35)$$

$$\neg \exists x(Living(x) \wedge Sp(x)) \rightarrow \neg \exists y FuN(y) \quad (36)$$

$$\exists x(Living(x) \wedge Sp(x)) \rightarrow \exists y \exists^{\geq 1} z(FuN(y) \wedge ReN(z)) \quad (37)$$

Characteristics of conditions

$$C(x) \rightarrow NASO(x) \tag{38}$$

$$VaRa(x) \rightarrow (Set(x) \veebar R(x)) \tag{39}$$

$$\neg\exists x Ax(x) \rightarrow \neg\exists y C(y) \tag{40}$$

$$\exists x \neg\exists y((Ax(x) \wedge VaRa(y)) \rightarrow \neg hasValues(x,y)) \rightarrow \neg\exists z C(z) \tag{41}$$

$$\begin{split}\forall x, y \exists z \exists^{=m} w((HV(x) \wedge C(y) \wedge conditionOf(y,x) \wedge elementOf(w,z) \wedge U(w) \\ \wedge(VaRa(z) \rightarrow Sp(z))) \rightarrow \neg(n > m) \wedge (n, m \in \aleph))\end{split} \tag{42}$$

$$\forall x \exists^{\geq 1} y((HV(x) \wedge HV_p(y)) \rightarrow ((y_i = y_j) \veebar (\neg(y_i = y_j) \wedge (y_i \cap y_j = (\emptyset \veebar \neg\emptyset)))) \tag{43}$$

$$\begin{split}\exists x(FuN(x_i) \wedge FuN(x_j) \wedge \neg(x_i = x_j)) \rightarrow \\ \exists y(HV(y_i) \wedge HV(y_j) \wedge has(x,y) \wedge \neg(y_i = y_j))\end{split} \tag{44}$$

$$\begin{split}\exists x(FuN(x_i) \wedge FuN(x_j) \wedge (x_i = x_j)) \rightarrow \\ \exists y(HV(y_i) \wedge HV(y_j) \wedge has(x,y) \wedge (y_i = y_j))\end{split} \tag{45}$$

$$\begin{split}\exists x, y, z((FuN(x_i) \wedge FuN(x_j) \wedge \neg(x_i = x_j) \wedge (has(x,y) \wedge HV(y) \wedge \\ conditionOf(z,y)) \rightarrow C(z) \wedge conditionOf(z,x_i) \wedge conditionOf(z,x_j)))\end{split} \tag{46}$$

$$HV_s(x) \rightarrow CN(x) \tag{47}$$

$$\neg(HV_p = HV_s) \tag{48}$$

Characteristics of, and related to, the *ReN*

$$Environment(x) \rightarrow \exists^{>1} y, z(C(y) \wedge C(z) \wedge Biotic(y) \wedge Abiotic(z) \wedge \neg(y = z)) \tag{49}$$

$$\exists x(Environment(x) \rightarrow \neg\exists y(Living(y) \wedge Sp(y)) \tag{50}$$

$$\exists x(Environment(x) \rightarrow \exists y(Living(y) \wedge Sp(y)) \tag{51}$$

$$\begin{split}\exists x((Environment(x_i) \wedge Environment(x_j) \wedge \neg(x_i = x_j)) \rightarrow \\ \exists y, z(Living(y) \wedge Sp(y) \wedge O(z) \wedge belongsTo(z,y) \wedge livesIn(z_i, x_i) \\ \wedge livesIn(z_j, x_j) \wedge \neg(z_i = z_j)))\end{split} \tag{52}$$

$$Environment(x) \rightarrow \exists^{\geq 1} y, z(ReN(y) \wedge Living(z)) \tag{53}$$

$$Habitat(x) \rightarrow NASO(x) \tag{54}$$

$$\neg\exists x(Habitat(x) \rightarrow \neg\exists y(Living(y) \wedge Sp(y))) \tag{55}$$

$$\exists x(Habitat(x) \rightarrow \exists y(Living(y) \wedge Sp(y))) \tag{56}$$

$$\begin{split}Habitat(x) \rightarrow \exists^{\geq 1} y, z(Living(y) \wedge Sp(y) \wedge O(z) \wedge belongsTo(z_i, y_i) \wedge \\ belongsTo(z_j, y_j) \wedge \neg(z_i = z_j) \wedge \neg(y_i = y_j) \wedge livesIn(z,x))\end{split} \tag{57}$$

$$Habitat(x) \rightarrow \exists^{\geq 1} y ReN(y) \tag{58}$$

$$(ReN(x) \wedge Habitat(y)) \rightarrow (partOf(x,y) \veebar (realisedBy(z,x) \wedge FuN(z) \wedge (Habitat(y) \leftrightarrow Ax(y)))) \tag{59}$$

$$(ReN(x) \wedge Habitat(y) \wedge partOf(x_i,y) \wedge partOf(x_j,y) \wedge \neg(x_i = x_j) \wedge realisedBy(z_i,x_i) \wedge realisedBy(z_j,x_j) \wedge FuN(z) \wedge \neg(z_i = z_j)) \rightarrow ((C_{z_i}(w_i) \cap C_{z_j}(w_j)) = \neg\emptyset) \tag{60}$$

$$ReN(x) \rightarrow (CN(x) \wedge NASO(x)) \tag{61}$$

Species and non-empty niches

$$Living(x) \rightarrow FuN(y) \tag{62}$$

$$Extinct(x) \rightarrow \neg FuN(y) \tag{63}$$

$$Living(x,t) \rightarrow PRE(x,t') \wedge PRE(x,t) \wedge (t' < t) \wedge Sp(x) \wedge T(t) \tag{64}$$

$$(instance(x,y,t) \rightarrow (O(x) \wedge Sp(y) \wedge T(t)) \tag{65}$$

$$Living(x) \leftrightarrow \neg Extinct(x) \tag{66}$$

$$ReN(x) \rightarrow (Living(y) \wedge Sp(y)) \rightarrow \exists z,w(Environment(z) \wedge Habitat(w)) \tag{67}$$

$$(Extinct(x) \wedge Sp(x)) \rightarrow \neg\exists y,z(Environment(y) \wedge ReN(z)) \tag{68}$$

4 Description of the axioms

4.1 About the definitions

1. The EN consists of a particular combination of the FuN (which implies the HV, C), Sp, ReN (implying HV_p) and *Living* (which implies O).
2. The FuN is defined as being a HV for a living species (i.e. not extinct).
3. The ReN is defined as being a HV_p of FuN's HV for a living species. In ReN, its hypervolume is smaller than the HV hypervolume. The 'physical realisation' has a space, exists in 3D and time, but *not* that it *is* a 3Dspace: it's a hypervolume that is smaller than the FuN-level hypervolume. One can either say a HV (and a HV_p) is an abstract region AR, or multidimensional objects are real entities, or ReN and its HV_p are $NASO$s, because e.g. 'realised niche' is a name that humans gave to it.
4. The hypervolume HV consists of multiple Cs, with unique conditions, and is an AR.
5. HV_p is a hypervolume like HV is and is a proper part of its HV.
6. C consists of a property/concept/entity, here just called the Ax, and its values (*quale*), and the relation between the latter two.
7. The axis of a C is a property or a concept/entity. The detail on the kinds of things that can make up see such an axis is flexible. The characteristics of the conditions are still under investigation. E.g. $NPOB$, Q, F.
8. A living (non-extinct) species has organisms at time t, where t denotes the present.
9. An extinct species had organisms at some time t in the past but not now. It is arguable if an extinct species is still present or not: if we, humans, have named it while there being no living organisms anymore, one could say that the species is present, but not the organisms of the species. Further, there are species extinct we do not know of

and/or have not named yet, so in some sense are 'not present', but this can be dismissed as an epistemological issue. Stretching it further, one can say that fossils are present so that there are 'dead organisms' present, hence requiring to specify the concept of what makes an organism living and what not, which goes into more detail than required for the purpose. Also, 'virtually extinct' is not elaborated on here.

4.2 Relations

10. EN constrains, on a 'meta' level the FuN.
11. Likewise, FuN is constrained by the constraints as specified at the level of EN.
12. For each species there are organisms that belong to – are categorised to be of type – that species.
13. HV_p is a proper part of the HV.
14. y is a condition of hypervolume x (and therefore this counts for the HV_p too).
15. That the *ReN* realises the FuN is debatable: it only *partially* realises the FuN.
16. See (15).
17. With (3) and (4), this also counts for the relation between the *ReN* and FuN.
18. This follows trivially from (17).
19. This follows from (3) and (28): *constrainedBy* is transitive. 20. W.r.t. (39): the elements of the set *Set* are entities (universals U). There are other restrictions between the elements of a set, like ((*Bread* and *Water*) or *Cheese*), not axiomatised yet.
21. HV_s is a part of HV.
22. To relate the HV to its FuN.
23. To relate the HV_p to its *ReN*.

4.3 Regarding other characteristics and constraints

24. EN is a CoNcept (and therefore has a definition DF [5]) and is a Non-Agentive Social Object. The $NASO$ can exist on the basis of agreement humans belonging to some community have regarding the concept; phrased differently: a $NASO$ exists because some humans conceived the idea. An AB can exist independent of the human mind that gives the name to it. Further, this human-concocted $NASO$ can 'contain', describe, an AB. In this interpretation, an ecological niche (at least EN and FuN) is a $NASO$ and the hypervolume is an AR, because the hypervolume-niche for a species was already there for ages, but only named as such last century (see also (3)).
25. FuN is a CoNcept and a Non-Agentive Social Object. See text of (21) for justification.
26. Organisms are APOs.
27. A species is a $NASO$. The species itself is not agentive, it's the organisms that belong to a species that are. Particular species, denoted with, for instance, *Atta sexdens* and *Mus musculus*, are still $NASO$s.
28. There can be multiple of those HV_p formed for each HV.
29. These HV_ps may have overlapping conditions but not necessarily so. Or stated differently: the HV_p can be, but do not have to be, exclusive.
30. Either the amount of conditions of a HV_p is less than that of the HV, or the values of one or more conditions are 'smaller' (a subsection) of its HV. The former opens up a can of worms regarding non-essential conditions of a fundamental niche: I'm looking into this.
31. Straightforward 1:n relation between FuN and *ReN*, that for each FuN there are

one or more ReNs.
32. For EN the FuN and its Sp are related.
33. If there's a relation between a species and a niche, and there's another relation between the same niche and a species, then the species are the same.
34. A species cannot have two (or more) FuNs. Put differently, if there are two niches and only one species, then the FuNs are the same.
35. Trivially, if there isn't a species, then there is no niche either.
36. Moreover, if x is not a living species, there is no niche for that species.
37. Stated differently, there must be a species and living organisms, i.e. alive, for a corresponding niche to exist, see also the discussion on the non-empty niches.
38. Condition C is a $NASO$.
39. $VaRa$ is a Set or can be some number interval (TR, PR and AR in DOLCE are intended for the values of the qualities, and are subsumed by R). The value range is either a set and the elements are values and are some EnDurant or PerDurant entity, or the actual values are numerical values between two extremes and the extremes do not have the same value. This is comprised in the DOLCE formalisation already.
40. If there's no Ax, then there's no C.
41. And if there is an Ax, but no values associated with it, then there's no C either.
42. The set of a HV contains the *maximum* amount of elements, i.e. there are m elements w (w_1, ..., w_m) and that is the maximum amount of elements (there are *not* n elements where $n > m$). From (20) and (13) follows that the value range of the axes of condition of the HV_p contains less elements. Further restrictions (to be investigated and formalised), expanding on (39): the values in an R and between the values of two different R (axes) can have further constraints on the values.
43. The multiple HV_ps that are proper parts of HV can have overlapping conditions, but not necessarily so.
44. With two different species, and each thus with a different FuN, then their corresponding HVs are distinct as well.
45. Vice versa, if the FuNs are the same, then their HVs are the same. Both this one and (44) follow trivially from (33) and (34). Note that, for epistemological reasons, a FuN, and therefore a HV may be *underspecified*, but what is assumed here, that each is *fully* specified.
46. But, in addition to (44), there are FuNs (and therefore ReNs too) for different species that have one or more overlapping conditions.
47. One or more conditions (but less than all conditions) of the HV can be grouped together, where each subset represents a concept, such as 'environmental parameters', 'lab conditions', or corresponding to the 'role' the organisms of a species play.
48. Just to be sure, the HV_p and HV_s are not the same thing (see also (5)).

For the ReN, some other aspects such as the environment and habitat have to be introduced, which are characterised here at a high level of granularity (i.e. simplified and only for what is required for/related to the niche).

Note that (some of) the parameters and their values of the environment are *used* when identifying (or searching for) a realised niche. One measures the values of some axis of a condition in the given environment *over time* and subsequently can use the resulting range as a $VaRa$ of an Ax of a C that is part of the HV_p of the ReN for the focal organisms. By doing some more research and *combining* the thus obtained value ranges for the particular parameter, it is possible to elucidate the C for the FuN of the species the focal organisms belong to.

49. *Environment* is a combination of biotic and abiotic conditions. This is extremely summarised but suffices for the task at hand.
50. There exist environments where no organisms live.
51. There are also environments where organism do lives.
52. That different organisms belonging to the same species live in different environments at the same time.
53. There are multiple (realised) niches of multiple species in some environment, i.e. a 1:n relationship.
54. The $Habitat$ is, like *Environment*, underspecified, but suffices for the task at hand. The habitat is a $NASO$.
55. There does not exist a habitat where nothing lives. Put differently:
56. In every habitat there are organisms living because there is no 'uninhabitable habitat'. And, more importantly:
57. The organisms (of a species) living in a habitat can belong to different species, or: organisms from different species live in a single habitat. Note that this is a major difference with the realized niche. In a shorthand notation:
58. There are multiple realised niches in a habitat, i.e. a 1:n relationship between the two.
59. The niche at the realised level is part of the habitat, but the habitat is some generalisation and not bound to one single spatio-temporal location, hence not possible to define as such in the strict sense. A habitat, e.g *CoastalWetlands*, can also be interpreted as a condition of the hypervolume, i.e. organisms and species live in a realised niche and $Habitat$ is one of the conditions that constitutes the HV of its species FuN.
60. In a habitat, conditions specified in one HV_p (i.e. that are part of a realised niche) overlap with conditions specified in another HV_p of a realised niche, where the two (or more) hypervolumes are realised hypervolumes from different species (hence from different hypervolumes).
61. The realized niche is a NASO (see (3), (21)).
62. A living species always has a fundamental niche.
63. Consequently, if the species is extinct, then there is no niche for that species.
64. A *species* is deemed to be alive if it used to be present some time ago, and is still present, plus that x has instances itself (i.e. organisms that are classified as belonging to species x) as well (which follows from (8)).
65. For all instances of O that belong to an instance of Sp, then this instance of O at time t is an organism categorised as belonging to some species. This is to avoid inconsistencies with unclassified organisms.
66. A trivial axiom that say that if a species belongs to the living, it is not extinct. Vice versa is arguable with relation to 'virtually extinct' species, which is not elaborated on here.

Additional species trivia can be added with respect to reproduction, genotype and phenotype, spatial locations, species differentiation, and can be extended with populations of organisms etc. While species are an essential component of the ecological niche, this is not elaborated on in this document, but has been separately formalised in accordance with the 'standard view' of species. NicheDiversification = the same species but with another niche at a later point in time. NicheDifferentiation = the 'species' differs that much, that it has become two different species, each with its own niche. NicheDiversification normally precedes NicheDifferentiation. Also, this is not elaborated on in this document (contact author for details).

Regarding non-empty niches: see §3.3.5 in "Ontologising bioscience: a niche" for a more detailed discussion. In short, an ecological niche *cannot* be empty by its very definition. For species conservation efforts, when biodiversity experts say they are looking for an "empty niche" this means that they are looking for an environment that falls within (has conditions and values equal or smaller than) the (endangered) species' FuN and the particular environment that 'matches' becomes an ReN *if and only if* the (re)introduced (organisms belonging to a) species survive in that particular environment.

67. Simpler version of (37), with the addition after the second implication, i.e. that then there is also some environment and habitat where the organism of a species live.
68. The converse, that if the species is extinct, then there is no environment and no ReN.

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