The granular perspective as semantically enriched granulation hierarchy

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Abstract: One can granulate data and information in multiple ways to generate granulation hierarchies. What the characteristics of such hierarchies are and what consequences they have on levels of granularity is, however, left implicit. We propose an explicit representation of such additional information of granulation hierarchies and transform them to *granular perspectives*. Granular perspectives can be uniquely identified, hence, distinguished, by means of formally representing their semantics using a granulation criterion and type of granularity used for granulation. The granular perspectives are equipped with both a simple relation and with mereological relations to consistently relate them, thereby facilitating cross-granular querying. Given the premises, definitions, and proven properties, consequences for characterising levels of granularity within such perspectives are demonstrated.

Keywords: granularity; granular perspective; granulation hierarchy; semantics of granularity; criterion of granulation; granular computing.

Reference to this paper should be made as follows: C.M Keet. (2011) 'The granular perspective as semantically enriched granulation hierarchy', *Int. J. Granular Computing, Rough Sets and Intelligent Systems*, Vol. x, No. x, pp.xxx–xxx.

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1 Introduction

The granulation of data, information, or knowledge results in granules, which are grouped into levels of granularity so that these levels can be organised in a granulation hierarchy. To generate such hierarchies, one can take a data-centric approach focussed on objects and their values by using rough and fuzzy sets and logic (e.g., Lin, 2009; Zadeh, 1997; Yao, 2004) as well as the more traditional crisp semantics using set theory or a first or higher order logic (e.g., Lin and Qing, 2007; Bittner and Smith, 2003), where a sound theory of the latter aids investigating details of the former (see e.g., Reformat and Ly, 2009). These approaches, however, do not reveal explicitly *how* we make or detect a granulation hierarchy, what its properties are, how one identifies the hierarchies, and how those hierarchies can be managed computationally in a consistent and reusable way in a granulated information system. The hierarchies we focus on here have levels such as cell $\prec$ tissue $\prec$ organ $\prec$ body and second $\prec$ minute $\prec$ hour, i.e., granulation of the subject domain[1] where objects, or the classes or concepts they instantiate, reside in such levels. There are few proposals to represent such hierarchies formally. For instance, lattices that can be a mere set of levels of detail (Stell and Worboys, 1998) or indicated as "multiviews" where each lattice presents a different perspective on the data (Chen and Yao, 2006). They do not have a means to represent what the properties are that are used to generate the lattices. Qiu *et al.* (2007) introduce the notion of "granular world" that corresponds to a level, and the union of such granular worlds is called a "full granular space", which corresponds to a granulation hierarchy that must be a taxonomy. However, they do not specify criteria for unifying the granular worlds. Bittner and Smith (2003) focus on hierarchies solely based on the parthood relation, but they did not propose a means to relate the hierarchies to each other. Domain experts, however, do want to link hierarchies in their information systems, such as in Geographic Information Systems (Camossi *et al.*, 2003; Keet, 2009), medicine (Grizzi and Chiriva-Internati, 2006; Ribba *et al.*, 2006; Keet and Kumar, 2005) and for ontologies to be used in data integration in the Semantic Web for Life Sciences (Smith *et al.*, 2007). A requirement is to perform conditional selections on levels across hierarchies, e.g.:

For a map at the Country-level of granularity,
show also the rivers with flow $\geq$ 100000 dm^3/minute.

The execution of rules across granularities is a similar requirement; e.g.:

If the doctor needs a daily *view of the growth of the cancer in patient#1,*
then deliver the tissue *samples*

as opposed to delivering cell cultures (or microarrays) at the Cell-level (Molecule-level, respectively) to monitor over intervals at the Hour-level (or Minute-level, respectively). Or one may want to query for all granulated properties, e.g.:

Retrieve all properties of Monocyte at the Cell-level.

To develop computational support to meet such user requirements, one has to be able to identify hierarchies so as to distinguish between them, after which one can relate the hierarchies consistently and transparently.

We introduce the concept of *granular perspective* in order to solve the user requirements to identify and link granulation hierarchies. A granular perspective provides a means to represent precisely and explicitly the hitherto implicit characteristics of granulation hierarchies. To arrive at this point, we take a formal, ontology-inspired, approach, with which we identify and prove several properties of granular perspectives. Mainly, it will be shown that the combination of criterion for granulation and the type of granularity determines uniqueness of a granular perspective, that levels in the perspective have the same type of granularity, and that each level resides in exactly one perspective. Decisions as to how one identifies the granular perspectives have consequences for a definition of granular level, which we shall address insofar as it follows from the characterisation of granular perspectives. With these characteristic in place, we can address how to link up the different granular perspectives and the levels that reside in such perspectives, hence, providing the formal foundation to solve the user requirements for cross-granular information retrieval. We introduce both a simple linking of the perspectives for ease of implementation and an ontologically-motivated one based on mereology. While it may be possible to argue about the chosen ontological commitments, the purpose is to demonstrate the consequences of such choices and propose a solution. The formal characterisation of the essential properties of a granular perspective is elegant and easy to implement in a modeling framework for specifying the declarative knowledge about the granulation and also usable for retrieving granulated information.

In the remainder of the paper, we first provide a brief overview in Section 2, which is followed by the characterisation of granular perspective and its basic relations in Section 3. Some consequences for granular levels are discussed in 4 and the linking of perspectives is addressed in Section 5. We conclude in section 6.

2 Informal overview

The principal entities, relations, and constraints that will be discussed and formalised in the following sections are depicted in the left-hand and top-half of Figure 1. The TypeOfGranularity that relates the types of granularity to perspective and level are only summarised in the figure. These types of granularity each describe different mechanisms of granulation, such as using the parthood relation, multi-representation of an object, semantic aggregations (including taxonomies), and aggregating by fixed calculations (60 seconds in a minute, etc.). It suffices for the current scope to know that the first distinction is made between quantitative (sG) and qualitative (nG) granularity; the reader is referred to (Keet, 2010) for the description of the rationale and explanation of the taxonomy. To ensure each granulation hierarchy is consistent ontologically, two properties have to be taken into account. The first ingredient is that exactly one of those types of granularity must be used to devise the levels in the hierarchy; this is depicted with the blob and line next to the rectangle labelled with has granulation. Consequently, the levels adhere to the same type of granularity as the perspective they reside in. The second ingredient is the Criterion for granulation with which one selects which properties of the objects are used to granulate and demarcate a section of the subject domain to generate the hierarchy. Both the type of granularity and the criterion are necessary for the identification of a granular perspective, which is denoted with the divided circle. The Criterion itself consists of at least two properties, one of which is a QualityProperty if the type of granularity used for the granular perspective is of the (quantitative) **sG** type. Due to space limitations, we shall not address all ontological considerations and justifications (such as why the precedes relation is at least a strict total order (Keet, 2007) with 1:1 participation constraint between the levels in a hierarchy (Keet, 2008)). The components in Figure 1 do not mention explicitly the data, information, or knowledge that is granulated, because it serves as an organising framework for granulation. It is beyond the current scope to discuss this in-depth, and we shall simply use the notion that they—particulars, universals, concepts, i.e., theory $\mathcal{T}$ or interpretation domain Δ—reside in a level.

3 Identifying granular perspectives

Most contributions to granular computing focus on generating granules and granular levels, although granulating a body of data, information, or knowledge invariably results in a granulation *hierarchy* that encompasses and orders the granules and levels. Little knowledge about the characteristics of such hierarchies is described formally as declarative knowledge and in a computationally easily reusable way across software applications. It is possible to extract various informal assumptions that ideally should be stated explicitly to make this implicit knowledge available for computational use, such as the mechanisms of granulation and that the objects' attributes have been used. Henceforth, we denote such a more comprehensively represented 'granulation hierarchy with additional properties' a *granular perspective*, abbreviated as *GP*. With the notion of granular perspective, one does not necessarily have to know which levels are in the perspective and how,

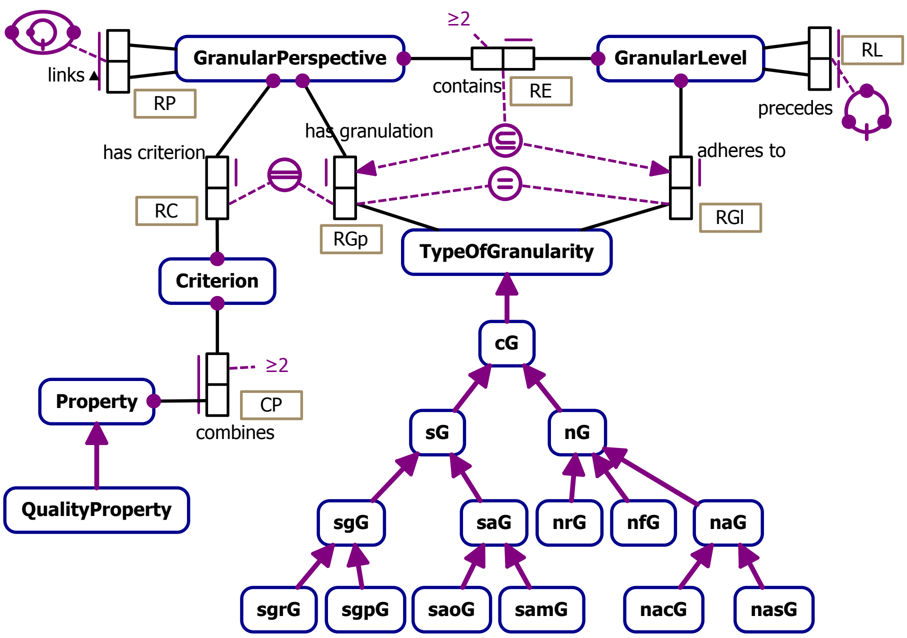

Figure 1 Graphical depiction of the main entity types (roundtangles), their relations (rectangles), and constraints (purple icons) for granularity; see text for explanation and formalisation.

but only that there are levels in the hierarchy. Take, e.g., the perspectives human structural anatomy, modes of transmission of infectious agents (Keet and Kumar, 2005), and administrative regions (Camossi *et al.*, 2003), which give an indication what is granulated. They do not indicate if human structural anatomy has a mere 4 levels of detail (e.g., body, organ, tissue, and cell) or ≥ 7 by also covering body part, organelle, and molecule to group the anatomical entities in a more fine-grained way. Instead, it indicates just that the perspective will have some levels. Observe that none of these perspectives mention other aspects of the entities that are granulated, such as the functions of anatomical entities, the mode of action of the infectious bacteria, and the geographical region of the cities, respectively. This approach assumes that they are covered by *other* perspectives like human functional anatomy, mode of action, and geographic regions, respectively. That is, when granulating entities, one highlights and *chooses a perspective* by using *one or more properties (attributes) along which to order the entities.* Normally, one does not use all represented properties of the entities to create a hierarchy with levels, although it is possible.

Thus, one can use one or more particular attributes and group its values at different levels of detail or use some other characteristic for construction and identification of a granulation hierarchy whilst ignoring other attributes. For instance, the human structural anatomy that, in that hierarchy, ignores other properties of those entities such as a cell's function and the organ's spatial location. This basic notion of 'highlighting', i.e, the usage of a selection of properties, has been noted elsewhere as well (Bittner and Smith, 2003; Chen and Yao, 2006). It requires, however, a closer ontological investigation into what kind of things those properties or attributes are, given that many kinds of properties have been identified by philosophers (Swoyer, 2000). Of particular interest are:

- Sortal property that provides principles of identity (e.g., being a chair);
- Essential property where the individual always has that property for the time of the individual's existence (being a dog is an essential property of Lassie);
- Natural (protein) and artificial (television) kinds;
- Attribute-like properties such as characterising properties that "do not divide the world up into a definite number of things" (e.g., being square redness requires a thing to be square-shaped or red, respectively);
- Primary property as objective feature of the world (size, protein conformation) versus relational properties such as the extrinsic one (married to) and secondary properties that "somehow depend on the mind" (taste).

It is clear that sortal and essential properties and artificial and natural kinds are more suitable for use in taxonomies and partonomies (using e.g., **nrG** or **naG** type of granularity), whereas the characterising and primary properties provide measurable characteristics of the objects, and therefore are easily usable for quantitative granularity (**sG**). However, more ontological investigation is needed to ascertain which of those kinds of properties are used for granulation and to constrain granulation to those kinds. For now, it suffices to observe that it is important that one *does* granulate according to specific properties with which the subject domain is partitioned.

Given this observation and looking ahead to computational use of granular perspectives, the requirement surfaces to formally represent it. Knowledge representation languages and programming languages are flexible about how to formally represent properties, such as attributes in a UML Class diagram or as unary or binary predicates. For our purpose, we can gloss over these finer details of implementations and focus on enabling the representation of the characteristics of granular perspectives. To this end, we generalise from this and capture it within a *criterion for granulation.*

3.1 The criterion for granulation

Each granular perspective has a criterion that provides the properties by which one performs the granulation. Winther (2006) and Chen and Yao (2006) allude to using criteria as well, albeit philosophically in an informal manner or formally but lacking an ontological foundation, respectively. I will characterise the criterion for granulation more precisely, but let us start with a few examples of granular perspectives and their criteria that are taken from the literature.

Example 3.1: Eight of the nine granular perspectives identified earlier by Keet and Kumar (2005) for the granulation of the infectious diseases domain had two components for the criterion for granulation (the remaining one needs refinement), such as Source of infection with a modifier Mode of transmission (p_1), Site with modifier Site of entry (p_2) or Site of effect (p_3), and Mode of action in pathology combined with Function as particular property for the modes of action of the infectious agent (p_6) versus pathological structure (p_7) and process (p_8). Other examples of criteria in biomedicine are Human structural anatomy and Cancer growth activity at different levels of granularity (Grizzi and Chiriva-Internati, 2006; Kumar *et al.*, 2005; Ribba *et al.*, 2006). Several ecosystem hierarchies have been analysed by Salthe (1985) who proposes a Genealogical hierarchy of nature that takes *time* into account and Ecological hierarchy of nature for energy exchanges in systems of *spatial* extent. Mota *et al.* (1995) aggregate ecological *processes* at different *time* granularities and Sorokine *et al.* (2006) combine ecological *units* with *scale*—i.e., also using two properties.

These examples do not suffice to make a criterion always a combination of two properties as general constraint for all granular perspectives. In addition, the criterion for granulation may have a sequence with a step-wise demarcation toward the focal property for granulation instead of properties of equal importance in the granulation. Further, the combination of properties is different for perspectives that have **nG** or **sG** types of granularity, because some scale is *always* involved in the criterion for quantitative, scale-dependent granularity. This, then leads me to propose that the criterion for granulation, C, is a combination of either at least two properties, $Prop$, or at least one property and a quality property, Q, where $\forall x(Q(x) \rightarrow Prop(x))$, that has a measurable region. The idea behind the distinction between $Prop$ and Q is to have a means to represent the difference between qualitative and quantitative granularity. For the latter (**sG** and its subtypes), the criterion C is a combination of a sortal or essential property or natural or artificial kind and a characterizing or secondary property (quality) for the value or value range that is determined by the type of scale used. For example, Surface (a $Prop$) with 'modifier' the Surface metric scale (a Q) measured in, say, m^2, dam^2, km^2—hence, with three levels l_1, l_2, and l_3 in that granular perspective—for **saoG** type of granularity. Thus, the semantics and usage of a scale for granulation is part of a granulation criterion, which is represented with QualityProperty.

Let us introduce some axioms and definitions, for which we assume to have a suitable first order language $\mathcal{L}$ with model-theoretic semantics. We use $has\_value(x, y)$ (Definition 3.2 and Proposition 3.3) for a means to record the values[2] (Proposition 3.4). Let us also make explicit the above-mentioned association between Q and **sG** (Proposition 3.5), which is rather weak now, but it will be refined later on.

Definition 3.2: (has_value) The *has_value* relation relates a property with its value: $\forall x, y(has\_value(x, y) \rightarrow Prop(x) \wedge V(y))$.

Proposition 3.3: Each quality property $Q(x)$ has some value $V(y)$, which is related through the relation $has\_value(x, y)$: $\forall x(Q(x) \rightarrow \exists y(has\_value(x, y)))$.

Proposition 3.4: By upward distributivity, value(s) of the property/ies $Prop$ and/or Q of the criterion are also values of the criterion C: $\forall x, y(has\_value(x, y) \rightarrow \exists z(has\_value(z, y) \wedge C(z)))$.

Proposition 3.5: If a criterion C has at least one $Prop$ and exactly one Q, then it is somehow associated with granulation type **sG**.

For qualitative types of granularity (**nG** and its subtypes in Figure 1), the criterion supplies the *category* to which the properties of the entities belong, e.g. processes, social entities, whereas the amount of specific properties of the individual entities considered at a finer-grained level increases (such as with taxonomic subsumption). In addition, the properties combined for a single criterion are less than or equal to the full combination of properties that are necessary for the universal or concept, or instances thereof. That is, any criterion C will not provide a single obvious property with changing numerical values for non-scale-dependent levels across the hierarchy. For instance, in the straightforward perspective of human structural anatomy, we have l_i = Organ and l_j = Cell without an obvious distinctive value other than a change in name (not using a measurement). Or take processes, where, say, Pathological process of infectious diseases is granulated according to **nrG**, we can have a granulation with processes and part-processes so that an entity Congestion, is involved in Inflammation. Congestion possibly can reside in some other granular perspective as well so that not all properties of Congestion are taken into account in this granular perspective.

We need a way to relate those properties that combine into a criterion it is used for, CP (Definition 3.6), and use that relation in a basic definition of criterion C in Definition 3.7.

Definition 3.6: (CP) The relation CP relates a criterion C to the properties it combines: $\forall x, y(CP(x, y) \rightarrow C(x) \wedge Prop(y))$, where at least two properties participate: $\forall x(C(x) \rightarrow \exists^{\geq 2} y\ CP(x, y))$

Definition 3.7: (Criterion) Each criterion C is a combination of either

- at least two properties $Prop$ but not a quality property Q, i.e., $\exists^{\geq 2} y(Prop(y) \wedge \neg Q(y))$, or
- at least one $Prop$ and exactly one Q, i.e., $\exists y \exists! z(Prop(y) \wedge Q(z) \wedge \neg(y = z))$.

which are related to C through the CP relation. More precisely:
$\forall x((C(x) \rightarrow \exists^{\geq 2} y(CP(x,y) \wedge Prop(y) \wedge \neg Q(y))) \veebar (C(x) \rightarrow \exists y \exists! z(CP(x,y) \wedge CP(x,z) \wedge Prop(y) \wedge Q(z) \wedge \neg(y = z)))$.

Both the process of selection of properties used for a particular granulation and how they are combined to form the granulation criterion is still somewhat underspecified and may benefit from further ontological investigation by philosophers, which is outside the scope of this paper. In any case, the criterion provides the *what* is to be granulated in addition to the the type of granularity (TypeOfGranularity, TG) that provides the *how*.

3.2 The granular perspective

The criterion and type of granularity have to be related to the granular perspective (GranularPerspective, GP) before defining it. The former is done with the relation RC (read: has criterion, Definition 3.8) and the latter through RG_p (read: has granulation, Definition 3.9) where the greek letters serve as syntactic sugar for the eight leaf types of granularity (i.e., a finite list of first order axioms so that it remains within first order logic).

Definition 3.8: (RC) Relation $RC(x, y)$ holds between perspective $GP(x)$ that has criterion $C(y)$: $\forall x, y(RC(x, y) \rightarrow GP(x) \wedge C(y))$.

Definition 3.9: (RG$_p$) The relation $RG_p(x, \phi)$ holds if $GP(x)$ and $TG(\phi)$ where TG is the type of granularity from the taxonomy of types of granularity (Keet, 2010): $\forall x, \phi(RG_p(x, \phi) \rightarrow GP(x) \wedge TG(\phi))$.

In addition to the typing of RC and RG_p, several constraints can be added. First, one can add an existential quantification to RC, because there is no reason to have a criterion for granulation in an information system without actually using it (Proposition 3.10). Second, one can neither use more than one criterion for one perspective nor use none, therefore Proposition 3.11 is added. The intuition of this proposition is that, ontologically, it is nonsense to combine, say, criterion $c_1 =$ Human pathological processes at different levels of granularity with $c_2 =$ Mouse structural anatomy at different levels of granularity to make one single hierarchy of levels. (No criterion amounts to selecting nothing or everything, which is no granulation.)

Proposition 3.10: Each criterion must participate in a RC: $\forall x(C(x) \rightarrow \exists y\ RC(y, x))$.

Proposition 3.11: Each perspective has exactly one criterion: $\forall x(GP(x) \rightarrow \exists! y\ RC(x, y))$.

Recollecting one must use a type of granularity for granulation, we obtain a mandatory participation of GP in the RG_p relation, because if one does not use a type of granularity at all, then one does not granulate as it would negate any granular structure among entities. Also, one should not mix different ways of granulation in one perspective lest the hierarchy of levels will be inconsistent. This is so because each type of granularity in the taxonomy is disjoint and the structure of the contents is different for each leaf type, hence combining two or more types leads to a contradiction. Thus, also for RG_p, there is exactly one TG for each GP:

Lemma 3.12: Each perspective has exactly one type of granulation: $\forall x(GP(x) \rightarrow \exists! \phi\ RG_p(x, \phi))$.

Visualising this set of constraints in Figure 1, we now have completed the path from Granular_Perspective (GP) through exactly one has criterion (RC) to Criterion (C)—which combines at least 2 Property ($Prop$)—and Granular_Perspective (GP) has granulation (RG_p) exactly one TypeOfGranularity (TG). Even with these basic relations and constraints, one can devise a simple definition that is easy to implement in information systems. Let D^f denote the entity that demarcates the granulation components for the granulated entities and that contains all the explicitly defined granular perspectives to granulate the subject domain, where RE is the relation between between D^f and GP (see below). In addition, we reuse the notions of concept (CN) and definition (DF) from the DOLCE foundational ontology (Masolo *et al.*, 2003). Then one can define GP as:

Definition 3.13: (Granular perspective (simple definition)**)** $\forall x \exists! w, y, z, \phi$ such that $GP(x)$ is a concept $CN(x)$, has a definition $DF(x, y)$, relates to its criterion $C(z)$ through the relation $RC(x, z)$, has granulation, RG_p, of type $TG(\phi)$ and is contained in a domain $D^f(w)$, i.e.: $\forall x(GP(x) \triangleq \exists w, y, \exists! z, \phi(DF(x, y) \wedge RC(x, z) \wedge RE(x, w) \wedge RG_p(x, \phi)))$.

Ontologically, more characteristics can be represented, but this comes at the cost of the need for a computationally more complex language, which is due to the identification constraint over the path between C and TG through GP. The additional identification constraint is particularly interesting for modelling with granularity, and therefore we will introduce it here as a constraint that can be added to the definition. To arrive at the point where we can prove the identification constraint holds, we start with Lemma 3.14, which strates that if one uses a Q in the criterion, then one uses scale-based granularity (**sG**).

Lemma 3.14: If $C(x)$ has a $Q(y)$ and $RC(z, x)$, then that $GP(z)$ has granulation type **sG**: $\forall x \exists z, \phi((C(x) \rightarrow \exists! y(CP(x, y) \wedge Q(y))) \wedge RC(z, x) \wedge RG_p(z, \phi) \rightarrow (\phi \rightarrow sG))$.

Proof: First, recollect Definition 3.7 and its main disjunction. Given we have a Q, then the second part after the exclusive-or in Definition 3.7 must hold. Second, we have the typing of RC (*Definition 3.8*) and existential quantification,
$\forall x(C(x) \rightarrow \exists y\ RC(y, x))$ (*Proposition 3.10*)
therefore, there has to be an instance, a, of GP (first argument in RC). Given this instance a, *Definition 3.9* of RG_p and
$\forall x(GP(x) \rightarrow \exists! \phi\ RG_p(x, \phi))$ (*Lemma 3.12*)
therefore, there must be a ϕ that is a TG (by the 'exactly one' [$\exists!$] and typing of RG_p). By having Q (first point) and Proposition 3.5, then $\phi = sG$, therefore $GP(z)$ has granulation type **sG**. □

It follows immediately from this proof that the first part of the definition of C applies to **nG** (Corollary 3.1), thanks to the exclusive-or in the definition of C and that the subtypes in the taxonomy of types of granularity are disjoint.

Corollary 3.1: If $C(x)$ has ≥ 2 properties $Prop(y)$ and $\neg Q(y)$, then $GP(z)$ has granulation type **nG**.

The interesting property of granular perspectives that really contributes toward identification, is the possibility to reuse a criterion provided the type of granularity is different (Lemma 3.15). From this proof it follows that the combination of criterion and type of granulation determines uniqueness of a GP (Theorem 3.16); that is, together they provide the necessary and sufficient conditions for identity of GP (this will be illustrated with an example afterward).

Lemma 3.15: A criterion C can be used with more than one perspective GP, provided the perspectives have distinct granulation types TG: $\forall x_1, x_2, y, \phi_1, \phi_2(RC(x_1, y) \wedge RC(x_2, y) \wedge RG_p(x_1, \phi_1) \wedge RG_p(x_2, \phi_2) \wedge \neg(x_1 = x_2) \rightarrow \neg(\phi_1 = \phi_2))$.

Proof: For each GP we have a $C(y)$ and a $TG(\phi)$, because of
$\forall x(GP(x) \rightarrow \exists! y\ RC(x, y))$ (*Proposition 3.11*)
$\forall x(GP(x) \rightarrow \exists! \phi\ RG_p(x, \phi))$ (*Lemma 3.12*)
Assume for some y, i.e., instance $c_1 \in C$, and some ϕ, there is the same instance of x, $p_1 \in GP$, i.e., $RC(p_1, c_1)$ and $RG_p(p_1, \phi)$ hold too. Let us reuse ϕ for some other perspective, $p_2 \in GP$, so that $RG_p(p_2, \phi)$ and assume $p_2 \neq p_1$ hold. Let us also reuse c_1 for some other perspective, $p_3 \in GP$, *i.e.*, $RC(p_3, c_1)$ and assume $p_3 \neq p_1$ hold. Then we have two cases:
(i) $p_3 = p_2$: then by Proposition 3.11 and Lemma 3.12 either $p_3 = p_2 = p_1$ (thus contradicting the assumptions $p_2 \neq p_1$ and $p_3 \neq p_1$) or there is an elusive property α to negate the equality. There is no α, hence, it must lead to identity of GP with C and TG. Thus,
$\forall x_1, ..., x_4, y_1, y_2, \phi_3, \phi_4(RC(x_1, y_1) \wedge RC(x_2, y_2) \wedge RG_p(x_3, \phi_3) \wedge RG_p(x_4, \phi_4) \wedge y_1 = y_2 \wedge \phi_3 = \phi_4 \rightarrow x_1 = x_2 = x_3 = x_4)$.
(ii) $p_3 \neq p_2$: then by Lemma 3.12, we have $RG_p(p_3, \phi')$ and $\phi \neq \phi'$, and by Proposition 3.11, we have $RC(p_2, c_2)$ and $c_1 \neq c_2$.
Thus, reuse of criterion c_1 with another TG, ϕ', is demonstrated in (ii) with p_3. □

Theorem 3.16: The *combination* of some $C(y)$ with a $TG(\phi)$ determines uniqueness of each $GP(x)$.

Proof: Follows from Lemma 3.15, point (i). □

Illustrating the idea of the proofs, one can have, say, a c_i = Mouse structural anatomy that can be granulated according to different mechanisms, such as by a partonomy (ϕ) and as a taxonomy (ϕ'), so that there are two different granular perspectives, p_1 and p_2. One can reuse (ϕ) with another criterion, say, c_j = Human structural anatomy to obtain a third perspective, p_3, but if we combine it again with Mouse structural anatomy, then we obtain the *same* perspective p_1. It trivially follows from

Lemma 3.15 and Theorem 3.16 that the perspectives are unique for a particular granulation system D^f. To do this, we have to introduce the relation between D^f and GP first, which is the topic of the next subsection.

3.3 Relating the domain framework, perspective and level with RE

From an ontology viewpoint, it is more appropriate to use the notion of D^f compared to a simple set of perspectives, and to explicitly relate that to the perspectives with the relation RE because D^f and GP (as well as Granular_Level, GL) are, conceptually, the frames wherein the data or knowledge is allocated during the granulation. It is also representationally more convenient, because one can represent more semantics of how they are related and put constraints on the relations as one pleases. In addition, considering relating a granular level to its perspective, we can use the same relation.

Several options to characterise RE can be argued for, which are, most commonly, (*i*) the data-centric set-theoretic one as if GP is a subset of D^f and GL of GP (the objects allocated into the levels may be, but not the intension at the type level), (*ii*) mereological using the (proper) parthood relation, or (*iii*) some other type of relation alike a *used_in* or *belongs_to*, which is rather ambiguous. Taking a closer look at parthood (Varzi, 2004), then it will have to be at least a *proper* parthood, because there is always more than one granular level in a perspective (Keet, 2007) (so there is always a remainder: at least one other level). In addition, taking the specification of types of part-whole relations (Keet and Artale, 2008), and recollecting that both GL and GP are concepts, and with DOLCE's concept CN subsumed by endurant ED (i.e., $CN \subseteq ED$ in DOLCE), then it could be parthood or containment. Containment fits well with a 'conceptual space' but less so with a portion of reality of a granular level. Taking the generic common denominator, the participating entity types delimit it to be proper parthood, $ppart\_of$. Therefore, RE is made a kind of $ppart\_of$ and typed with D^f, GP, and GL as relata. Given that parthood is transitive, then if a GL is in a GP and a GP is in a D^f, the GL is in the D^f. The transitivity in the other direction, the *has_ppart* relation, is valid with the restriction that it holds for the default case of one granulation domain with one or more perspectives. This brings us to the definition for RE, where proper parthood is defined in terms of parthood in the usual way (i.e., $\forall x, y(ppart\_of(x, y) \triangleq part\_of(x, y) \wedge \neg part\_of(y, x))$).

Definition 3.17: (RE) For all x there exists a y where the relation $RE(x, y)$, and its inverse RE^-, holds between two of the three granularity components iff

- $GL(x) \wedge GP(y)$ or $GP(x) \wedge D^f(y)$ for $RE(x, y)$ and
- $D^f(x) \wedge GP(y)$ or $GP(x) \wedge GL(y)$ for $RE^-(x, y)$, with

$\forall x, y(RE(x, y) \rightarrow ppart\_of(x, y))$ and $\forall x, y(RE^-(x, y) \rightarrow has\_ppart(x, y))$.

With this definition, preceding analysis, and proper parthood, we can demonstrate that RE has the properties of being acyclic and transitive (Lemma 3.18). Acyclic means that an object x does not have a path to itself, More precisely, let φ be a variable ranging over relations, then a cyle can be written as (Eq. 1) and acyclicity is represented as $\forall x \neg\varphi(x,x)$.

$$\forall x_1,...,x_i,...,x_n(\varphi(x_1,x_i) \rightarrow (\varphi(x_1,x_2) \wedge ... \wedge \varphi(x_{n-1},x_n) \wedge (1 \leq i \leq n) \rightarrow x_1 = x_i)) \tag{1}$$

Lemma 3.18: *RE and RE^- are acyclic and transitive.*

Proof: We first demonstrate transitivity and then acyclicity. Given
$\forall x,y(RE(x,y) \rightarrow ppart\_of(x,y))$ (*from Definition 3.17*)
and from Ground Mereology that *ppart_of* is irreflexive, asymmetric, and transitive (Varzi, 2004), therefore RE is transitive as well. Acyclicity of RE, *i.e.*, the negation of (Eq. 1) where φ is substituted with RE, holds, because of (i) the domain and range restrictions on RE (Definition 3.17) that prohibits a $RE(x,y)$ where $D^f(x)$ and $GL(y)$, or, say, $GP(x)$ and $GP(y)$, and (ii) identity of the domain and range such that $\neg(x_1 = x_i)$ can be shown because instances of GL, GP, and D^f are distinct domain elements in any interpretation thanks to their definitions. This argument holds also for the inverse relation RE^-, where $RE^-(x,y) \rightarrow has\_ppart(x,y)$, and the typing of domain and range restrictions of RE^-. □

With the RE relation in place, we now can return to where we left off at the end of Section 3.2, i.e., that it follows from Lemma 3.15 and Theorem 3.16 that the perspectives are unique for a particular granulation system:

Corollary 3.2: Granular perspectives are unique within the domain they are contained in: $\forall x_1,...,x_n,y(GP(x_i) \wedge D^f(y) \wedge RE(x_i,y) \rightarrow \neg(x_1 = x_2) \wedge ... \wedge \neg(x_{n-1} = x_n))$.

Thus, all perspectives $p_1 \ldots p_n \in GP$ contained in a D^f are disjoint. One cannot derive a complete coverage unless one takes a closed-world assumption and assume that all entities in the represented subject domain must be granulated. However, at this stage, I do not make such a limiting commitment to the closed-world assumption only, which therefore still leaves the option to add it to an implementation and to enforce it.

Observe that Corollary 3.2 does not exclude the possibility to have two or more versions of "p_1" where the amount of levels in the perspectives are distinct, but then they reside in *different* granulation systems. This problem, which is a special case of standard data integration, is outside the scope of this paper and is discussed briefly in Chapter 5 of (Keet, 2008).

This concludes the initial characterisation and means for identification of granular perspectives. This affects the notion of granular level and what information about levels one can represent, as well as how the perspectives can be related to each other. For clarity of presentation, the former will be dealt with in the next section and the latter postponed to Section 5.

4 Preliminaries of granular levels

Compared to a mere granulation hierarchy that poses no constraints on the granular levels, the characteristics of a granular perspective do affect what it contains. Analogous to GP, we can say that a *granular level* (GL) is 'something more' than merely an arbitrary collection of granules. The specification of a particular level of granularity in a subject domain makes sense only after knowing the criterion and type of granulation, which are provided by the GP. This leads to the observation that if one has a granular level, then there *must* be a perspective it is contained in, lest one creates levels freely by combining types of granularity or mixing criteria that would result in inconsistent granulation. To ensure the level and perspective are related, we can reuse RE introduced above and add an existential quantification on RE for the level (Proposition 4.1).

Proposition 4.1: For all x, where $GL(x)$, x is contained in a granular perspective: $\forall x(GL(x) \rightarrow \exists y(RE(x,y) \wedge GP(y)))$.

This axiom can be constrained further by availing of indistinguishability and similarity so that the perspective must have at least two granular levels in a granular perspective ($\forall x(GP(x) \rightarrow \exists^{\geq 2} y(RE^{-}(x,y) \wedge GL(y)))$ (see Theorem 1 in Keet (2007)), because if there were only one level in the perspective, there is no granulation into coarser- and finer-grained details. With the relation between GL and GP established, we do not have to redefine the criterion for granulation for each granular level anymore, because this is already taken care of by GP's criterion C. However, the values of GP's criterion are needed to distinguish between different levels in a perspective and to establish that no two levels are identical in one granular perspective (i.e., each level can occur only once in a perspective). As for the type of granularity, the level clearly must adhere to the same type of granularity as its perspective. These properties of a granular level are consequences of both the nature of granular perspective and notions such as indistinguishability and similarity (Keet, 2007), but it does not preclude one from identifying and adding more properties or attributes to the notion of granular level. To arrive at such a basic, yet expandable, definition for GL, we first add a relation for GL that it also relates to a type of granularity, TG, which we realise with the adheres to relation, abbreviated in the formalisation with RG_l (Definition 4.2). It has an existential quantification to ensure the type of granularity constrains the structure of the contents of that level (Proposition 4.3).

Definition 4.2: **(RG_l)** The relation $RG_l(x,\phi)$ holds if $GL(x)$ and $TG(\phi)$, i.e., $\forall x,\phi(RG_l(x,\phi) \rightarrow GL(x) \wedge TG(\phi))$.

Proposition 4.3: Each GL must adhere to a TG: $\forall x(GL(x) \rightarrow \exists \phi\ RG_l(x,\phi))$.

With the addition of RG_l, we have sufficient ingredients to provide a basic version of a definition for granular level. GL delimits what it is to be a level and of a certain level and, analogous to GP, has a definition and constraints, and is a concept, too. Hence, we reuse several categories from DOLCE (Masolo *et al.*, 2003) again, being concept CN, definition DF, quality Q, and region V, and the previously introduced $has\_value(x, y)$ (Definition 3.2) and $RE(x, y)$ (Definition 3.17) are reused.

Proposition 4.4: (Granular level (preliminary version)) $\forall x \exists! v, w, y, z \exists p$ such that $GL(x)$ is a concept $CN(x)$, has a definition $DF(x, y)$, is related to $GP(w)$ with $RE(x, w)$ and uses criterion $C(z)$ with $RC(w, z)$ and $has\_value(z, v)$ where the value is in region $V(v)$ for any $GL(x)$ that *adheres_to* **sG**, $GL^s(x)$, and z's label for any $GL(x)$ that *adheres_to* type **nG**, $GL^n(x)$. Entities residing in $GL^s(x)$ are similar to each other with respect to (the value z of) $V(v)$, entities residing in $GL^n(x)$ are similar to each other with respect to (the label of the universal of) $Prop(p)$ of $C(z)$, and both are φ-indistinguishable with respect to its adjacent coarser-grained level; i.e., $\forall x(GL(x) \triangleq \exists! v, w, y, z(DF(x, y) \wedge GP(w) \wedge RE(x, w) \wedge C(z) \wedge RC(w, z) \wedge R(v) \wedge has\_value(z, v)))$.

Building upon this basic definition and the above-defined and proven characteristics, we can prove several additional properties. The so-called "role subset" (encircled "$\subseteq$") and "role equality" (encircled "$=$") constraints shown in Figure 1 will be proven first, which enforce that the perspective and the levels it contains have the same type of granularity:

$$\forall x, y(GP(y) \wedge GL(x) \wedge RE(x, y) \rightarrow \exists! \phi(RG_p(y, \phi) \leftrightarrow RG_l(x, \phi))) \tag{2}$$

We prove (2) in two steps: Lemma 4.5 alone does not ensure GP and its GL use the *same* TG because the "$\exists \phi$" says there is *at least one* of them, but to achieve it is the same, we need Lemma 4.6.

Lemma 4.5: For each $GP(x)$ and $GL(y)$ over their join paths, the following holds: if $GP(x)$ contains $GL(y)$, then $GP(x)$ has granulation some TG and $GL(y)$ adheres to some TG:

$$\forall x, y(RE(x, y) \wedge GP(y) \wedge GL(x) \rightarrow \exists \phi(RG_p(y, \phi) \wedge RG_l(x, \phi))) \tag{3}$$

Proof: First, given
$\forall x(GL(x) \rightarrow \exists y(RE(x, y) \wedge GP(y)))$ (*Proposition 4.1*)
$\forall x(GP(x) \rightarrow \exists^{\geq 2} y(RE^{-}(x, y) \wedge GL(y)))$ (*Theorem 1 in (Keet, 2007)*)
therefore, if we have a GP, then there must be ≥ 2 instances of GL related to it and if we have a GL that there must be a GP. Assume a, b such that $GP(a)$ and $GL(b)$, then with
$\forall y(GP(y) \rightarrow \exists! \phi \; RG_p(y, \phi))$ (*Lemma 3.12*)
$\forall x(GL(x) \rightarrow \exists \phi \; RG_l(x, \phi'))$ (*from Proposition 4.3*)
either $\phi = \phi'$ or $\phi \neq \phi'$ so that there must be ≥ 1 TG and therefore (3) holds. □

Lemma 4.6: For each TG, some $GL(x)$ adheres to that TG if and only if some $GP(y)$ has a granulation RG_p that TG: $\forall \phi(\exists y \; RG_p(y, \phi) \leftrightarrow \exists z \; RG_l(z, \phi))$.

Proof: Assume GP and GL are (mutually dependent) instantiated so that they must have a TG (Lemma 4.5). Given Lemma 3.12 and that each structure of level contents of the leaf types are distinct, then also $\forall x(GL(x) \rightarrow \exists! \phi \; RG_l(x, \phi'))$ must

hold, because combining two or more types leads to a contradiction. Further, from Proposition 4.4 we have "uses criterion $C(z)$..." and by
$\forall x(GP(x) \rightarrow \exists! y\ RC(x,y))$ (*Proposition 3.11*)
RE relating GL to its GP, having
$\forall x(GP(x) \rightarrow \exists! y, \phi(RC(x,y) \wedge RG_p(x,\phi)))$ (*Theorem 3.16*)
and aforementioned Lemma 3.12, therefore, the GL uses the same criterion as its GP, hence $\phi = \phi'$ holds, too. □

With these results, we can prove that each GL is contained in *exactly one* GP:

Theorem 4.7: For all x, where $GL(x)$, x is contained in *exactly one* granular perspective: $\forall x(GL(x) \rightarrow \exists! y RE(x,y))$.

Proof: We already have at-least-one GL in GP (Proposition 4.1) and need to demonstrate the at-most-one $(RE(x,y) \wedge RE(x,z) \rightarrow y = z)$. GL uses the C of GP it is contained in (Proposition 4.4), which still permits a GL to be reused in another GP. However, GL adheres to the same TG as its GP it is contained in (Eq. (2)). Given
$\forall x_1,...,x_4,y_1,y_2,\phi_3,\phi_4(RC(x_1,y_1) \wedge RC(x_2,y_2) \wedge RG_p(x_3,\phi_3) \wedge RG_p(x_4,\phi_4) \wedge$
$y_1 = y_2 \wedge \phi_3 = \phi_4 \rightarrow x_1 = x_2 = x_3 = x_4)$ (*Theorem 3.16*)
$\forall x_1,...,x_n,y(GP(x) \wedge D^f(y) \wedge RE(x,y) \rightarrow \neg(x_1 = x_2) \wedge ... \wedge \neg(x_{n-1} = x_n))$
(*Corollary 3.2*)
there cannot be another GP with the same C and TG in one D^f, hence, GL can be ≤ 1 time in a perspective. Thus, ≥ 1 and ≤ 1 is exactly one, *i.e.*, $\forall x(GL(x) \rightarrow \exists! y RE(x,y))$ □

With the declarative approach, it is in fact not that difficult to proceed further with an assessment if one can add more properties to GL. For instance, if the type of granularity permits or requires additional properties of granular levels. For qualitative granularity, one can take a closer look at possible and permissible *granulation relations*; that is, what the types of relations are that can hold between an object in a finer-grained granule and its level and an(other) object in the adjacent coarser-grained level. For quantitative granularity, the values of a level's usage of criterion is more encompassing or has a larger range than that of its adjacent finer-grained level for those levels that adhere to **sG** type of granularity. It is also possible to relate a function to such granular levels to be used for 'converting' contents of one level into its adjacent coarser level or vice versa—e.g., 60 * 1 minute = 1 hour—and that there are ≤ 2 mathematical functions associated to a granular level to take care of the conversions between these values; the formalisation and proofs of these simple additions can be found in (Keet, 2008).

5 Linking perspectives and levels

We now have the basic machinery to address linking granular perspectives and their levels so as to solve the problem of linking hierarchies, like those mentioned in the introduction with examples in GIS and medicine. Two strategies are possible, being exploiting mereology to overcross perspectives and levels, and chaining levels through the relation between levels, RL (not elaborated on here), and between perspectives, RP, that is depicted with the links relation in Figure 1. The two options are schematically depicted in Figure 2. The 'simple' RP relation can be typed with the perspectives as relata (Definition 5.1), from which follows that RP is irreflexive and symmetric (Lemma 5.2).

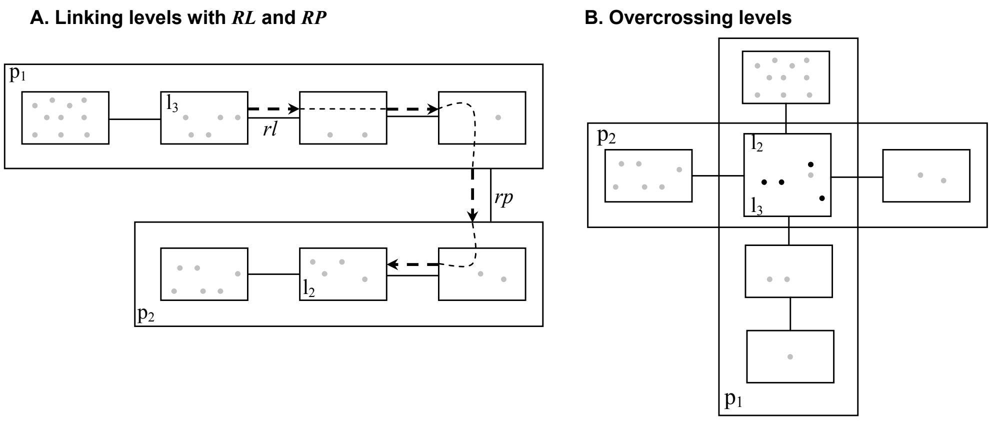

Figure 2 Connecting levels and perspectives with RL and RP (A) or overlap and overcross (B); grey dots represent the entities in the levels, of which the dark grey ones overlap (i.e., an intersection on the sets of objects residing in the levels is not empty).

Definition 5.1: (RP) RP relates two distinct perspectives:
$\forall x, y(RP(x, y) \rightarrow GP(x) \wedge GP(y) \wedge \neg(x = y))$.

Lemma 5.2: RP is irreflexive, $\forall x \neg RP(x, x)$, and symmetric, $\forall x, y(RP(x, y) \leftrightarrow RP(y, x))$.

Proof: Irreflexive: the "$\neg(x = y)$" in Definition 5.1 and one or more unique perspectives (Corollary 3.2), therefore the relata can never be the same. Symmetric: RP's distinct domain and range are both of type GP. □

This relation already can deal with representing which perspectives have to be linked, like a p_i = Body sample with the p_j = Time units in $RP(p_i, p_j)$ but not with a p_k = Human functional anatomy that are all three declared ina a granulated information system. One also can retrieve additional targeted information through using RP, which is not possible with the mereology-based strategy (discussed next) and it relies more on an overall framework for granularity. Let us take as example a d_i^f = Infectious Diseases (Keet and Kumar, 2005), Vibrio cholerae located at the Species-level l_7 in perspective p_1 = Linnaean Taxonomy and in l_3 = Inhibitor of a p_2 = Pathological mode of action. Then, using RL and RP, one can pick up information along the path to retrieve more knowledge by taking advantage of granularity to a greater extent; *in casu*, that at the coarser-grained l_1 of p_1, *V. cholerae* is a Bacterium and of the pathology p_2 in level l_1 a Toxin-producer. RP together with the perspectives

Table 1 Sample granular perspectives for cartography with conditional levels across perspectives (based on Camossi *et al.* (2003)) that link human geography with physical geography.

Admin (π_1)	$\overset{RP}{\Leftrightarrow}$	**Hydro** (π_2) (river with flow $\geq$)
Country	$\Leftrightarrow$	100 000 litres/min
$\uparrow$		$\uparrow$
Province	$\Leftrightarrow$	10 000 litres/min
$\uparrow$		$\uparrow$
Region	$\Leftrightarrow$	2500 litres/min
$\uparrow$		$\uparrow$
Municipality	$\Leftrightarrow$	1000 litres/min
$\uparrow$		$\uparrow$
Municipality district	$\Leftrightarrow$	250 litres/min

and their levels also make it easier to deal with the conditional perspectives as mentioned in the introduction, which is illustrated in the next example.

Example 5.3: Assume one wants to find correlations between the built environment and freshwater availability. To generate a cartographic map, the user needs a granular perspective of administrative entities and of hydrology. They are illustrated in Table 1, where π_1's type of granularity is nrG and criterion Administrative region, and π_2's type of granularity is sgpG and criterion River water throughput. The information retrieval has become trivial: if one selects the Municipality-level, then rivers with $\geq$ 1000 liters/min will be selected automatically thanks to RP. One may want to simplify this in an application (e.g., to increase performance) by declaring that each instantiation of RP holds at a specified level.

The second option is more elaborate and ontologically more precise. One can *overcross* perspectives, which means that the two levels are different, but they share at least some of their contents that thus *overlap* (depicted in Figure 2-B). Overlap and overcross have their usual semantics based on Ground Mereology (or an extension thereof) with *part_of* as primitive relation (Varzi, 2004):

$$\forall x, y(overcross(x, y) \triangleq overlap(x, y) \wedge \neg part\_of(x, y)) \quad (4)$$

$$\forall x, y(overlap(x, y) \triangleq \exists z(part\_of(z, x) \wedge part\_of(z, y))) \quad (5)$$

Concerning conents of levels, recollect the notion of contents residing in a level (Section 2), which we denote with the *in_level* relation between the contents and a level. We now can demonstrate that perspectives can overcross.

Lemma 5.4: *Two levels in different perspectives can* overcross*:*
$\forall x, y(overcross(x, y) \wedge GL(x) \wedge GL(y) \wedge \neg(x = y) \rightarrow \exists v, w(RE(x, v) \wedge RE(y, w) \wedge \neg(v = w)))$

Proof: The proof goes in two steps: first the "$\neg part\_of(x, y)$" of *overcross* is addressed, subsequently the "$overlap(x, y)$" part of *overcross*, where *overcross* is defined as in (4).

1. From typing RE (Definition 3.17), $GP(v)$ and $GP(w)$, with $\neg(v = w)$, therefore $\neg(x = y)$, because the combinations of criterion and granulation are distinct for the two levels (Theorem 3.16, Lemma 2); hence, $\neg part\_of(x, y)$ of *overcross* holds.
2. Demonstrate $overlap(x, y)$, defined as in (5). This applied to the axiom in the lemma implies that $GL(x)$ and $GL(y)$ must have a common part z. This is true if the *content* of a level stands in some part-whole relation to the frame that encloses the entities (/types) of the subject domain, because then the intersection of the contents of the two levels return the common part, which is z. Let the two sets with the levels' contents be denoted with X and Y, then $X \cap Y = z$ and $z = \neg\emptyset$. Given that the entities (/types) are not structural parts of granular levels, it will have to be a type of *containment* for overlap to hold. The *contained_in* relation

$$\forall x, y(contained\_in(x, y) \triangleq part\_of(x, y) \wedge V(x) \wedge V(y) \wedge \exists z, w(has\_3D(z, x) \wedge has\_3D(w, y) \wedge ED(z) \wedge ED(w))) \quad \textit{(Keet and Artale, 2008)}$$

is a subrelation of $part\_of$ and satisfies this idea but not the relata, only $part\_of$ and $ppart\_of$ do. Given that the same entity (/type) can be in different levels and thus shared among ≥ 1 whole—but not in the same hierarchy—it has to be $part\_of$. Then, because $\forall x, y(in\_level(x, y) \rightarrow part\_of(x, y))$ and $X \cap Y = z$, therefore $overlap(x, y)$ holds.
Both 1 and 2 return true, and thereby the levels can overcross. □

We can extend it to overcrossing perspectives as follows.

Theorem 5.5: *If two levels in different perspectives overcross, then their perspectives overcross:* $\forall x_1, x_2, y_1, y_2(overcross(x_1, x_2) \wedge GL(x_1) \wedge GL(x_2) \wedge GP(y_1) \wedge GP(y_2) \wedge RE(x_1, y_1) \wedge RE(x_2, y_2) \rightarrow overcross(y_1, y_2))$.

Proof: Given that

$$\forall x, y(RE(x, y) \rightarrow ppart\_of(x, y)) \quad \textit{(Definition 3.17)}$$
$$\forall x, y(ppart\_of(x, y) \rightarrow part\_of(x, y)) \quad \textit{(Varzi, 2004)}$$

the parthood relations are transitive in Ground Mereology, and so is RE (Lemma 3.18), then the overcross from Lemma 5.4 implies the respective perspectives of the levels overcross. □

Thus, we can have, say, entity A that is granulated with criterion c_1 in p_1 resulting in A' in a l_i in p_1 and is granulated with c_2 for p_2, allocated to l_j as A''. Overcrossing p_1 with p_2 and l_i with l_j ties A' to A'', providing the intersection where the properties of the entity combine to represent the property-rich A; hence, a richer representation of that entity than in their separate perspectives. Example 5.6 illustrates this for bacteriocins.

Example 5.6: Let d_i be the domain of Bacteriocins, which are non-therapeutical antibiotics used in food science and the food industry to improve food safety and preservation. Tn5301 is the gene encoding for the bacteriocin Nisin. Tn5301 is in level l_3 at the Gene-level in perspective p_1, and Tn5301 is in the Mobile DNA fragment-level and subsumed by the entity type Transposon in level l_2 of a location perspective (p_2). Overcrossing the two levels where the Tn5301s match, says that gene Tn5301 is on a transposon. Thus, the overlap provides a richer description of Tn5301, because it combines more properties the entity type has than is represented with only one perspective.

Although Example 5.6 and Figure 2-B demonstrate overcross for two perspectives and levels, this can be any amount of relevant levels and perspectives. This has been worked out in more detail for examples in geographic information systems (Keet, 2009) and for biological material entities independently by Vogt (2010).

Overall, the overcross option has less representational overhead compared to asserting instances of RP between the perspectives, but it is expected that it requires more computation during query execution.

6 Conclusions

We have proposed a way to formally represent a granular perspective as an enhancement to bare granulation hierarchies by providing a means to identify each perspective within a domain by the unique combination of criterion and its type of granularity used for granulation. We also have demonstrated some consequences that such granular perspectives have on a characterisation of a level of granularity that resides within such a granular perspective. Principally, those levels must adhere to the same type of granularity as their perspective and each level resides in exactly one perspective. Moreover, these enhancements make it transparent to link perspectives in a usable and reusable way so as to realise more complex granular analyses, which was identified as a requirement by domain experts. Two proposals for the linking of granular perspectives were made, being a simple relation between the perspectives and a more elaborate way that relies on mereology.

We are currently investigating how the theory can be implemented most efficiently so as to enhance the granulated information systems with the representation of and reasoning over granular perspectives.

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Notes

[1]i.e., not granularity with respect to the data-information-knowledge abstraction levels (Yao, 2009), which can be orthogonal to granulation of the subject domain.

[2]Note the value's upward distributivity from property to its criterion and that $has\_value(x, y)$ corresponds in spirit to "*ql*" in DOLCE foundational ontology of Masolo *et al.* (2003).