Propositional Logic – Lab 3

Marijke Keet

KRDB Research Centre, Faculty of Computer Science
Free University of Bozen-Bolzano, Italy
keet@inf.unibz.it

November 9, 2007

Entailment

- Truth value assignment (interpretation) of all atoms in Σ is a function $\mathcal{I}$ where $\mathcal{I} : \Sigma \rightarrow \{T, F\}$
- An interpretation $\mathcal{I}$ is a model of ϕ, written as $\mathcal{I} \models \phi$.
- We can do the same for *sets* of formulas Θ; i.e., $\mathcal{I} \models \Theta$ iff $\mathcal{I} \models \phi$ for all $\phi \in \Theta$
- We want formula ϕ to be implied by Θ, if ϕ is true in all models of Θ, written as $\Theta \models \phi$, so we get $\Theta \models \phi$ iff $\mathcal{I} \models \phi$ for all models $\mathcal{I}$ of Θ
- Properties of entailment.
  - Deduction theorem: $\Theta \cup \{\phi\} \models \psi$ iff $\Theta \models \phi \rightarrow \psi$
  - Contraposition theorem: $\Theta \cup \{\phi\} \models \neg\psi$ iff $\Theta \cup \{\psi\} \models \neg\phi$
  - Contradiction theorem: $\Theta \cup \{\phi\}$ is unsatisfiable iff $\Theta \models \neg\phi$

Entailment

1. E.g. Θ is knowledge base, KB, which contains $(A \vee C) \wedge (B \vee \neg C)$, and we want to know if formula ϕ holds, where $\phi = A \vee B$, i.e. $KB \models \phi$?
   - Hint: check all possible models: ϕ must be true wherever KB is true.... truth table.
2. $\models \neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$
3. $\models \neg(A \rightarrow B) \leftrightarrow (\neg A \vee B)$
4. $\models (A \rightarrow (B \wedge C)) \rightarrow (\neg(B \wedge C) \rightarrow \neg A)$
   - Hint: you can do it with truth tables, but have a look at the equivalences, too.

Equivalence

1. Which one(s) of the following is (are) equivalent?
    - $(((A \rightarrow B) \rightarrow B) \rightarrow B)$ and $(A \rightarrow B)$
    - $(A \land B) \lor C$ and $(A \rightarrow \neg B) \rightarrow C$
        - hint: for $(A \land B) \lor C$, try distributivity, or for $(A \rightarrow \neg B) \rightarrow C$, try implication
        - which gives: $(A \lor C) \land (B \lor C)$, resp. $\neg(A \rightarrow \neg B) \lor C$
        - more implication gives $\neg(\neg A \lor \neg B) \lor C$
        - hint: De Morgan.
        - $(\neg\neg A \land \neg\neg B) \lor C$
        - Double negation, then $(A \land B) \lor C$

From natural language to PL

1. Suppose that a island is inhabited by exactly two persons, Angelo and Roberto. If Angelo shaves an inhabitant then this shaves Roberto; moreover, if Roberto shaves an inhabitant then he does not shave Angelo. Therefore Roberto does not shave himself.
2. (i) Formalise the essential facts in propositional logic
3. Suppose that a island is inhabited by exactly two persons, Angelo and Roberto. If Angelo shaves an inhabitant then he shaves Roberto; moreover, if Roberto shaves an inhabitant then he does not shave Angelo. Therefore Roberto does not shave himself.

Properties

1. $((A \rightarrow B) \land (C \rightarrow \neg D)) \rightarrow (C \rightarrow \neg B)$
2. (ii) use truth tables [and/or] entailment [and/or] equivalences to establish whether the shaving argument is valid
3. (iii) provide a counterexample in case it is not, and relate it to the shaving story.

- brief introduction tableaux