Propositional Logic – Lab 4

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Summary

- A sound and complete procedure deciding satisfiability is all we need, and the **tableaux method is a decision procedure which checks the existence of a model**
- It exhaustively looks at all the possibilities, so that it can eventually prove that no model could be found for unsatisfiable formulas.
- $\phi \models \psi$ iff $\phi \land \neg\psi$ is NOT satisfiable—if it is satisfiable, we have found a counterexample
- Decompose the formula in top-down fashion

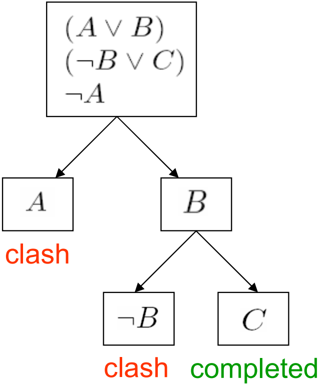
Basic rules

- Tableaux calculus works only if the formula has been translated into **Negation Normal Form**, *i.e.*, all the negations have been pushed inside
- Recollect the list of equivalences, apply those to arrive at NNF, if necessary.
- If a model satisfies a conjunction, then it also satisfies each of the conjuncts
- If a model satisfies a disjunction, then it also satisfies one of the disjuncts. It is a non-deterministic rule, and it generates two alternative branches.
- Apply the completion rules until either (a) an explicit contradiction due to the presence of two opposite literals in a node (a clash) is generated in each branch, or (b) there is a completed branch where no more rule is applicable.

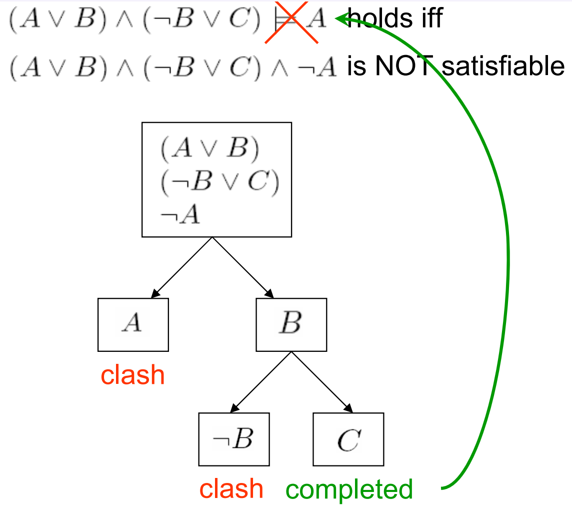
Example

$(A \vee B) \wedge (\neg B \vee C) \models A$ holds iff

$(A \vee B) \wedge (\neg B \vee C) \wedge \neg A$ is NOT satisfiable



Example


(A ∨ B) ∧ (¬B ∨ C) ⊨ A holds iff
(A ∨ B) ∧ (¬B ∨ C) ∧ ¬A is NOT satisfiable
(A ∨ B)
(¬B ∨ C)
¬A
A
B
clash
¬B
C
clash
completed

Exercises Tableaux

1. From Lab 2:
   - $\models \neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$
   - $\models (A \rightarrow (B \wedge C)) \rightarrow (\neg(B \wedge C) \rightarrow \neg A)$
   - check them against your truth tables, the answer should be the same (valid, contradiction, ...)
2. From Lab 3:
   - $(A \wedge B) \vee C \models (A \rightarrow \neg B) \rightarrow C$ (equivalences exercise)
   - $((A \rightarrow B) \wedge (C \rightarrow \neg D)) \rightarrow (C \rightarrow \neg B)$ (the shaving story)