First Order Logic – Lab 1

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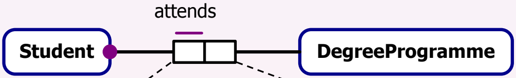
Tableaux in PL

1. From Lab 3:
   - $(A \wedge B) \vee C \models (A \rightarrow \neg B) \rightarrow C$ (equivalences exercise)
   - $((A \rightarrow B) \wedge (C \rightarrow \neg D)) \rightarrow (C \rightarrow \neg B)$ (the shaving story)

From data to ORM2 or text and then to FOL—or v.v.

Student **is an entity type**.
DegreeProgramme **is an entity type**.
Student attends DegreeProgramme.

Each Student attends **exactly one** DegreeProgramme.
It is possible that more than one Student attends **the same** DegreeProgramme.
OR, in the negative:
For each Student, **it is impossible that that** Student attends **more than one** DegreeProgramme.
It is impossible that any Student attends **no** DegreeProgramme.

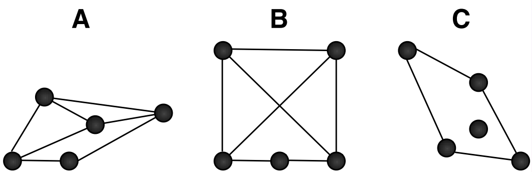


Attends

Student	DegreeProgramme
John	Computer Science
Mary	Design
Fabio	Design
Claudio	Computer Science
Markus	Biology
Inge	Computer Science

Examples of first-order structures

- Graphs are mathematical structures.
- A graph is a set of points, called **vertices**, and lines, called **edges** between them. For instance:



- Figures A and B are different depictions, but have the same descriptions w.r.t. the vertices and edges. Check this.
- Graph C has a property that A and B do not have. Represent this in a first-order sentence.
- Find a suitable first-order language for A (/B), and formulate at least two properties of the graph using quantifiers.

Checking

- Consider a first order language where R is a binary relation symbol and P a unary relation symbol (UML class, ER entity type, ORM object type) and an interpretation $\mathcal{I}$ with domain $\{0, 1\}$, where:

$$P^{\mathcal{I}} = \{0, 1\} \tag{1}$$
$$R^{\mathcal{I}} = \{(0, 0), (0, 1)\} \tag{2}$$

- Check whether $\mathcal{I}$ is a model of the following formulas:

$$\forall x \exists y R(x, y) \tag{3}$$
$$\exists x \forall y R(x, y) \tag{4}$$
$$\forall x P(x) \tag{5}$$
$$\exists x P(x) \tag{6}$$
$$\forall x \forall y (R(x, y) \vee P(x)) \tag{7}$$
$$\forall x \forall y (R(x, y) \wedge (P(x) \vee P(y))) \tag{8}$$