First Order Logic – Lab 4

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Midterm comments

- Take care of the **details** and a final answer is not enough. You need to demonstrate—**prove**—why it is that answer
- Tableaux

Tableaux summary (1/2)

- A sound and complete procedure deciding satisfiability is all we need, and the **tableaux method is a decision procedure which checks the existence of a model**
- It exhaustively looks at all the possibilities, so that it can eventually prove that no model could be found for unsatisfiable formulas.
- $\phi \models \psi$ iff $\phi \land \neg\psi$ is NOT satisfiable—if it is satisfiable, we have found a counterexample
- Decompose the formula in top-down fashion

Tableaux summary (2/2)

- Tableaux calculus works only if the formula has been translated into **Negation Normal Form**, *i.e.*, all the negations have been pushed inside
- Recollect the list of equivalences, apply those to arrive at NNF, if necessary.
- If a model satisfies a conjunction, then it also satisfies each of the conjuncts
- If a model satisfies a disjunction, then it also satisfies one of the disjuncts. It is a non-deterministic rule, and it generates two alternative branches.
- Apply the completion rules until either (a) an explicit contradiction due to the presence of two opposite literals in a node (a clash) is generated in each branch, or (b) there is a completed branch where no more rule is applicable.

First order logic: equivalences

- Those from PL + New in FOL: see lecture slides p30. In particular,
  $\neg\forall x\phi(x) \equiv \exists x\neg\phi(x)$
  $\neg\exists x\phi(x) \equiv \forall x\neg\phi(x)$
- The $\forall$ definition: $\forall x\phi(x)$ if and only if $\neg\exists x\neg\phi(x)$
- Just like with tableaux for PL, we may need them for rewriting a formula suitable for a FOL tableaux

Equivalences

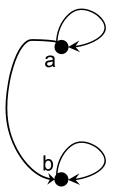
1. Rewrite $\neg\exists x \forall y (P(x) \rightarrow Q(y))$ into its negation normal form
2. Simplify $\neg\exists x \neg(P(x) \vee Q(x)) \wedge \forall x(P(x) \rightarrow Q(x))$
3. Are these equivalent/valid? prove it
   - $(\neg\forall y \neg P(y) \vee P(c)) \rightarrow (\forall y \neg P(y) \rightarrow P(c))$
     where P is a unary relation symbol and c a constant symbol
   - $\exists x \phi(x) \wedge \exists x \psi$ and $\exists x(\phi(x) \wedge \psi)$
     note that φ, $\exists x \varphi$, and $\forall x \varphi$ are provably equivalent, and x does not occur as a free variable of ψ

Tableaux

- $\{\forall x P(x), \exists x(\neg P(x) \lor \neg P(f(c)))\}$
- Lesser of two evils (compared to not reading the book): have a look at http://en.wikipedia.org/wiki/User:Tizio/Tableau_FO

More graphs

Consider the following graph, and first-order language $\mathcal{L} = \langle R \rangle$, with R being a binary relation symbol (edge).



1. Formalise the following properties of the graph as $\mathcal{L}$-sentences: (i) (a, a) and (b, b) are edges of the graph; (ii) (a, b) is an edge of the graph; (iii) (b, a) is not an edge of the graph. Let T stand for the resulting set of sentences.
2. Prove that $T \cup \{\forall x \forall y R(x, y)\}$ is unsatisfiable using tableaux calculus.