First Order Logic – Lab 5

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First order logic

The lexicon of a first order language contains:

- Connectives & Parentheses: $\neg$, $\rightarrow$, $\leftrightarrow$, $\wedge$, $\vee$, (and);
- Quantifiers: $\forall$ (universal) and $\exists$ (existential);
- Variables: $x, y, z, ...$ ranging over particulars;
- Constants: $a, b, c, ...$ representing a specific element;
- Functions: $f, g, h, ...$, with arguments listed as $f(x_1, ...x_n)$;
- Relations: $R, S, ...$ with an associated arity.

First order logic: equivalences

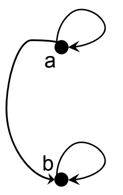
- Those from PL + New in FOL: see lecture slides p30.
- The $\forall$ definition: $\forall x \phi(x)$ if and only if $\neg \exists x \neg \phi(x)$
- Just like with tableaux for PL, we may need them for rewriting a formula suitable for a FOL tableaux

First order logic: tableau

- Finds a model for a given collection of sentences in negation normal form.
- Refutation tree, completion rules, apply rules until (a) an explicit contradiction (clash) or (b) there is a completed branch and no more rule is applicable (a completed branch gives a model of KB: the KB is satisfiable)
- Completion rules for quantified formulas:
  - If a model satisfies an existentially quantified formula, then it also satisfies the formula where that quantified variable has been substituted with a new *skolem constant*: for $\exists x \phi(x)$ we get, e.g., $\phi(c)$
  - If a model satisfies a universally quantified formula, then it also satisfies the formula where the quantified variable has been substituted with *some term*: for $\forall x \phi(x)$ we get, e.g., $\phi(a)$

Tableaux: More graphs

Consider the following graph, and first-order language $\mathcal{L} = \langle R \rangle$, with R being a binary relation symbol (edge).



1. Formalise the following properties of the graph as $\mathcal{L}$-sentences, and using variables: (i) (a, a) and (b, b) are edges of the graph; (ii) (a, b) is an edge of the graph; (iii) (b, a) is not an edge of the graph. Let T stand for the resulting set of sentences.
2. Prove that $T \cup \{\forall x \forall y R(x, y)\}$ is unsatisfiable using tableaux calculus.

More tableaux

- One of the following formula is valid, which one?
  1. $\exists y \forall x (F(y) \rightarrow F(x))$
  2. $\neg \exists y \forall x (F(x) \rightarrow F(y))$
- Try to guess first (recollect subsumption), then prove using tableaux and give a counterexample for the other one.
- Check your solution with the Tree Proof Generator at http://www.umsu.de/logik/trees/
- Consider the following arguments from the Kelly textbook:
  1. All fruit is tasty if it is not cooked. This apple is not cooked. Therefore it is tasty.
  2. All fruit is tasty if it is not cooked. This apple is cooked. Therefore it is not tasty.
- Determine which is valid by using the tableau calculus, and which is falsifiable by showing a counterexample.

Tarski's world

- Lets you experiment with interplay between PL or FOL sentences and interpretations, construct models, formalising NL and/or blocks in the blocks world to PL or FOL sentences, truth value assignments, validity, contradiction etc.
- CSLI Software, installed on the lab computers
  - Demonstration of the basics
  - Exercises (printouts)