README file for the OWL-based theories, accompanying: Orchestrating a Network of Mereo(topo)logical Theories

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1 CONTEXT

Parthood is used widely in ontologies across subject domains. Some modelling guidance can be gleaned from Ontology, yet it offers multiple mereological theories, and even more when combined with topology (i.e., mereotopology). To complicate the landscape, decidable languages put restrictions on the language features, so that only fragments of the mereo(topo)logical theories can be represented, yet during modelling, those full features may be needed to check correctness. We address these issues by specifying a structured network of theories formulated in multiple logics that are glued together by the various linking constructs of the Distributed Ontology Language, DOL. For the KGEMT mereotopological theory and its five sub-theories, together with the DL-based OWL species and first- and second-order logic, this network in DOL orchestrates 28 modular ontologies. Each module can be sent to its suitable automated reasoner.

This readme file contains a brief overview of the mereological, topological, and mereotopological theories you can find in this folder at http://www.meteck.org/files/ontologies/mereotopo/ and on OntoHub at https://ontohub.org/repositories/mereotopology/. First, there are the theories from Ontology, then a table with the 28 theories.

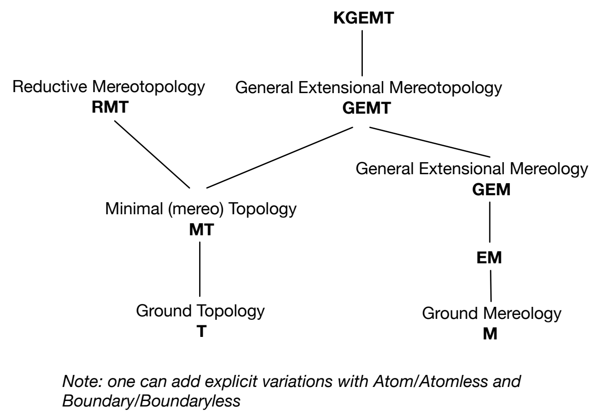

Figure 1: Hasse Diagram of the main mereo- and mereotopological theories; from weaker to stronger, going uphill (after descriptions in Varzi (2007)).

2 MEREOTOPOLOGICAL THEORIES

This summary of mereotopology is based on [5] and [3], which focus on going from the basic axioms making up the simplest theories of parthood and location up to the KGEMT mereotopological theory, as depicted in Fig. 1.

Starting with mereology, the basic theory is Ground Mereology, M, where part of is primitive, which is reflexive, antisymmetric, and transitive (t1, t2, t3 in Table 1). With this, one can define proper part of (t20 in Table 2), from which irreflexivity (t25) can be deduced, and, following from that (antisymmetry+irreflexivity), asymmetry (t27); proper part is also transitive (t26). Overlap can now also be defined (t21). Among the many things one can add to M, there's the notion of supplementation, eventually resulting in General Extensional Mereology, GEM (M + t4, t5).

In the other section, we begin with Ground Topology (T) with the connection relation, which is reflexive and symmetric (t6, t7). This can be extended to Minimal Topology (MT) by adding t8 to it regarding spatial enclosure, where enclosure is defined as in t9. This MT is then combined with GEM to be GEMT, which consists of MT + GEM + t10 (self-connected), t12 (bridging connection to part), and t13 (fusion). With the GEMT axioms and definitions, one can then define interior proper part (t24), and from that, tangential proper part (t23). The final aspect is then about closure, interior, and exterior, resulting in KGEMT, i.e., GEMT + t14, t15, t16. The three extra axioms require their definitions (t17-t19), so they then also belong to KGEMT.

It surely is possible to construct a mereotopological theory in a different way. For instance, one could take proper parthood as primitive, or merge parthood and location into a ternary relation, or adding atomicity or boundaries (see [5] for details). One can add more on topology (e.g., [4]), and consider containment [1] and convex hulls [2]. The scope here is just one of those sets of interrelated theories, not all conceived ones.

3 MATCHING THEORIES TO LOGICS

Given these six theories, and considering OWL, OWL 2, full first order logic and second order logic, then we come to the representable theories as listed in Table 3. The first 19 also have been represented in their respective OWL files available at the aforementioned URLs. We took, one by one, a theory listed in Fig. 1 with the axioms that make it up (see previous section), and assessed which of those that can be represented in each of the languages. Note that not every combination of the 27 axioms in Tables 1 and 2 make sense ontologically: e.g., a theory consisting of t1 (reflexivity of parthood) and t16 (exterior) is not a recognised mereo(topo)logical theory, so therefore no such theory is listed in the table. The converse—just

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Table 1: Axiomatization of KGEMT core axioms and definitions (based on [3], summarised from [5]). P: partof; PP: proper part of; O: overlap, C: connection; E: enclosure; EQ: indiscernibility; IPP: interior proper part; TPP: tangential proper part; SC: self-connected; c: closure; i: interior; e: exterior; +: sum; ~: complement.

$P(x,x)$	(t1)
$P(x,y) \land P(y,z) \to P(x,z)$	(t2)
$P(x,y) \land P(y,x) \to x = y$	(t3)
$\neg P(y,x) \to \exists z(P(z,y) \land \neg O(z,x))$	(t4)
$\exists w \phi(w) \to \exists z \forall w(O(w,z) \leftrightarrow \exists v(\phi(v) \land O(w,v)))$	(t5)
$C(x,x)$	(t6)
$C(x,y) \to C(y,x)$	(t7)
$P(x,y) \to E(x,y)$	(t8)
$E(x,y) =_{df} \forall z(C(z,x) \to C(z,y))$	(t9)
$E(x,y) \to P(x,y)$	(t10)
$SC(x) \leftrightarrow \forall y,z(x = y + z \to C(y,z))$	(t11)
$\exists z(SC(z) \land O(z,x) \land O(z,y) \land \forall w(P(w,z) \to (O(w,x) \lor O(w,y)))) \to C(x,y)$	(t12)
$z = \sum x \phi x \to \forall y(C(y,z) \to \exists x(\phi x \land C(y,x)))$	(t13)
$P(x, cx)$	(t14)
$c(cx) = cx$	(t15)
$c(x + y) = cx + cy$	(t16)
$cx =_{df} \sim (ex)$	(t17)
$ex =_{df} i(\sim x)$	(t18)
$ix =_{df} \sum z \forall y(C(z,y) \to O(x,y))$	(t19)

Table 2: Basic additional axioms, definitions, and theorems (based on [3], summarised from [5]).

$PP(x,y) =_{df} P(x,y) \land \neg P(y,x)$	(t20)
$O(x,y) =_{df} \exists z(P(z,x) \land P(z,y))$	(t21)
$EQ(x,y) =_{df} P(x,y) \land P(y,x)$	(t22)
$TPP(x,y) =_{df} PP(x,y) \land \neg IPP(x,y)$	(t23)
$IPP(x,y) =_{df} PP(x,y) \land \forall z(C(z,x) \to O(z,y))$	(t24)
$\neg PP(x,x)$	(t25)
$PP(x,y) \land PP(y,z) \to PP(x,z)$	(t26)
$PP(x,y) \to \neg PP(y,x)$	(t27)

the six named theories in Fig. 1—does not apply either, because even ground mereology cannot be represented fully in OWL 2 DL (in short: no antisymmetry). Finally, one cannot assert property definitions alike t20 (see Table 2) in OWL whereas one can in FOL and HOL, and therefore they were added as primitives.

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Table 3: Subsets of KGEMT that can be represented in HOL, FOL, and the OWL species. For the OWL species, t9, t20, t21, t22, t23, t24 were simplified and added as primitives (axiom number of Tables 1 and 2 are appended with a "p"). For readability, FOL and HOL are not listed where OWL species are listed, and OWL 2 DL is not listed if it lists an OWL 2 DL fragment.

N	Language	Subsets of KGEMT axioms	Comments
1	OWL 2 QL	t1, t21p, t22p	M, with p, partially
2	OWL 2 QL	t6, t7	T, c
3	OWL 2 QL	t20p, t21p, t22p, t25, t27	M, pp
4	OWL 2 QL	t6, t7, t8, t9p	MT
5	OWL 2 QL	t1, t6, t7, t8, t9p , t10, t20p, t21p, t22p, t23p, t24p, t25, t27	GEMT, partial
6	OWL 2 EL, 2 QL	t1, t2, t21p, t22p	M, with p, partially
7	OWL 2 EL	t6	T, c, partial
8	OWL 2 EL, 2QL	t6, t8, t9p	MT, partially
9	OWL 2 EL	t1, t2, t6, t8, t9p, t10, t26, t20p, t21p, t22p, t23p, t24p	GEMT, partial
10	OWL 2 RL, OWL Lite, DL	t2, t21p, t22p	M, p, partial
11	OWL 2 RL, 2QL, OWL Lite, DL	t7	T c, partial
12	OWL 2 RL, EL, DL, OWL Lite, DL	t2, t26, t20p, t21p, t22p	M, with p and pp both partially
13	OWL 2 RL, OWL Lite, DL	t7,t8, t9p	MT partial
14	OWL 2 RL, OWL Lite, DL	t2, t7, t8, t9p, t10, t26, t20p, t21p, t22p, t23p, t24p	GEMT, partial
15	OWL 2 DL	t1, t2, t6, t7, t8, t9p, t10, t25, t27, t20p, t21p, t22p, t23p, t24p	GEMT, partial
16	OWL 2 DL	t1, t2, t6, t7, t8, t9p, t10, t26, t20p, t21p, t22p, t23p, t24p	GEMT, partial
17	OWL 2 RL	t2, t20p, t21p, t22p, t25, t27	M with p and pp, both partial
18	OWL 2 DL	t1, t2, t25, t27, t20p, t21p, t22p	M with p and pp, both partial
19	OWL 2 EL	t1, t2, t26, t20p, t21p, t22p	M with p and pp, partial
20	FOL, HOL	t1, t2, t3, t21, t22, t4	M, with p
21	FOL, HOL	t1, t2, t3, t20, t21, t22, t25, t26, t27	M, with p and pp
22	FOL	t1-t4, t20, t21, t22, t25, t26, t27	GEM, partial
23	FOL, HOL	t6, t7, t8, t9	MT
24	FOL	t1-t10, t12, t20-t27	GEMT, partial
25	FOL	t1-t4, t6-t10, t12, t20-t27, t14-t19	KGEMT, partial
26	HOL	t1-t5, t20, t21, t22, t25, t26, t27	GEM
27	HOL	t1-t10, t12, t13, t20-t27	GEMT
28	HOL	t1-t10, t12, t13, t20-t27, t14-t19	KGEMT